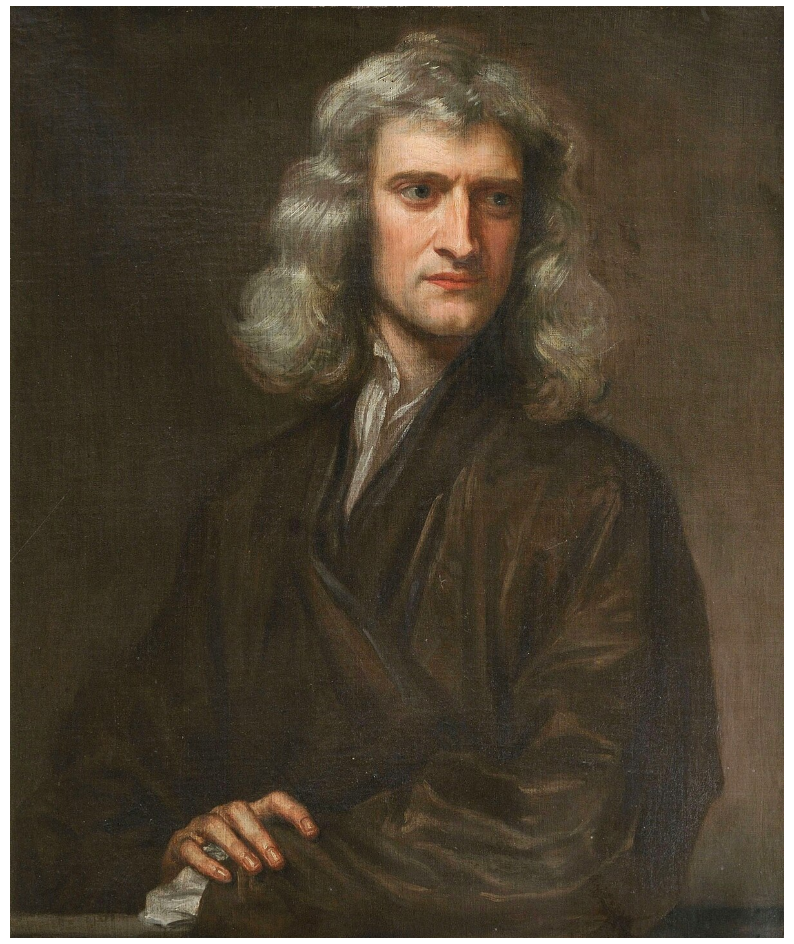
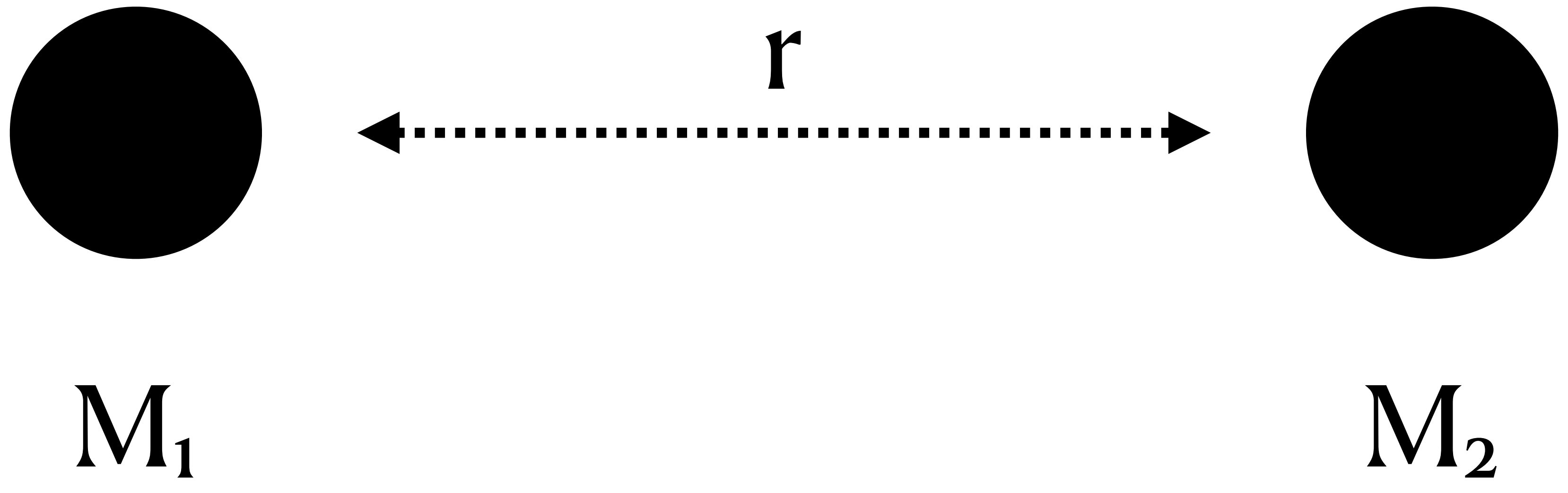
Testing Gravity and Quantum Mechanics

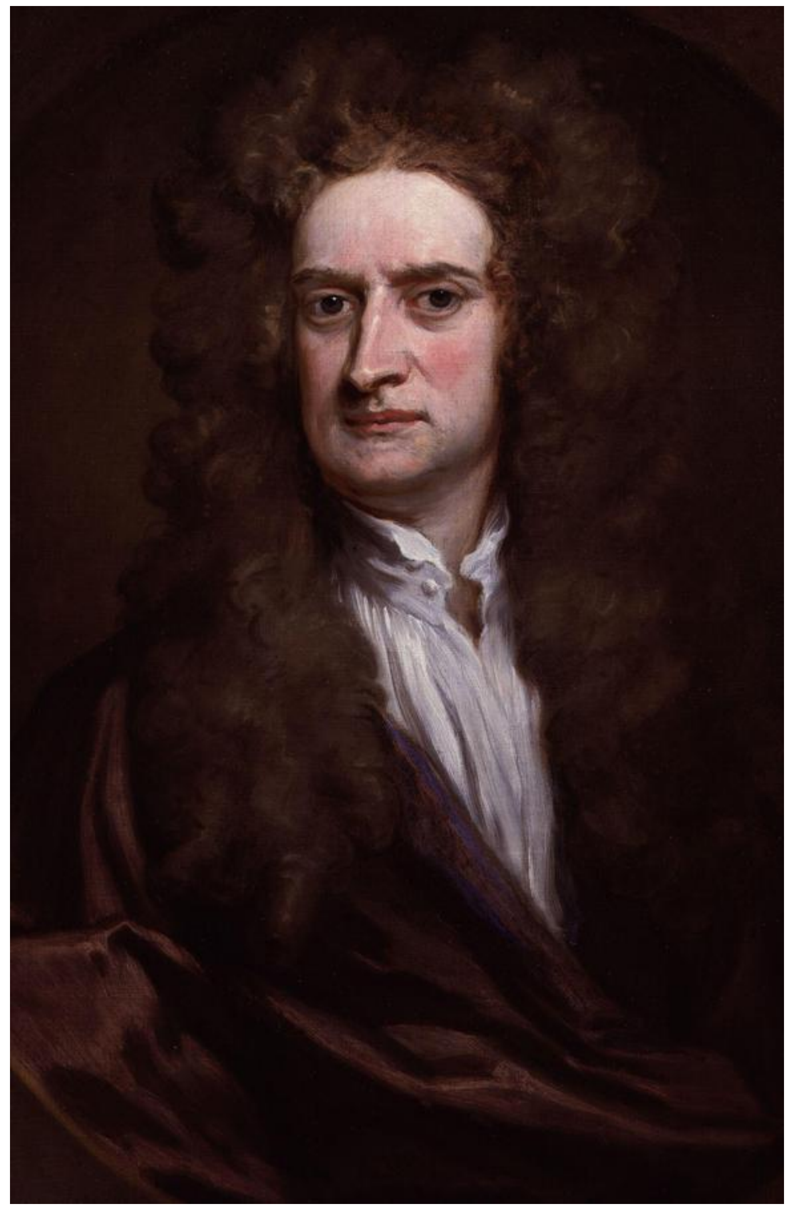
Surjeet Rajendran

Gravity

Testing Gravity







$$F = \frac{GM_1M_2}{r^2}$$

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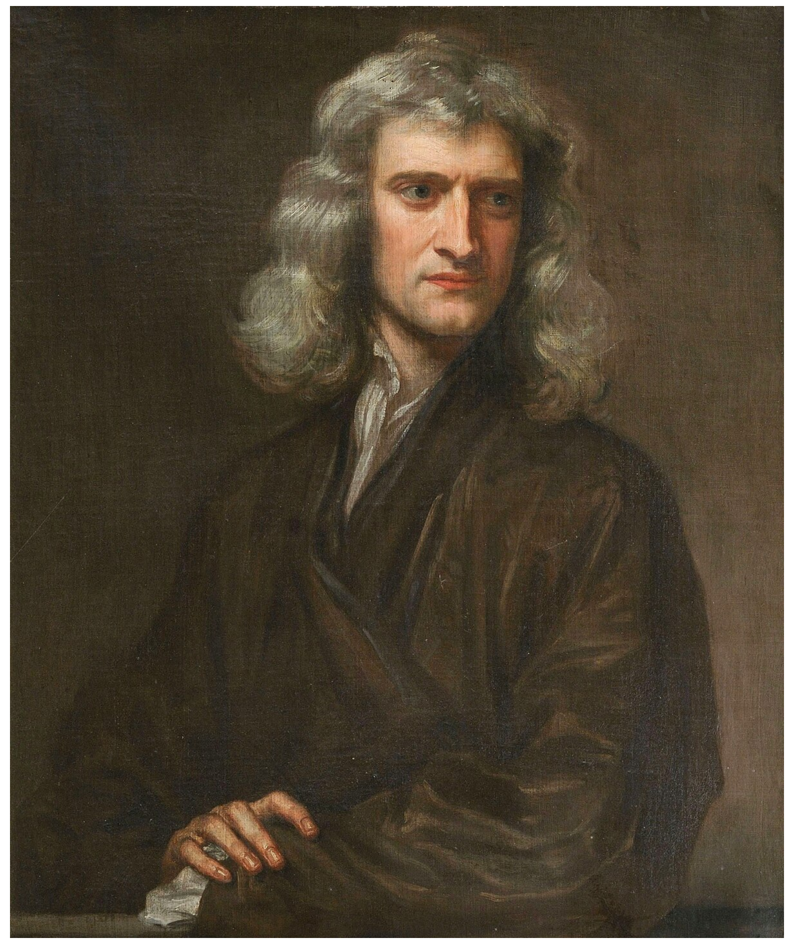

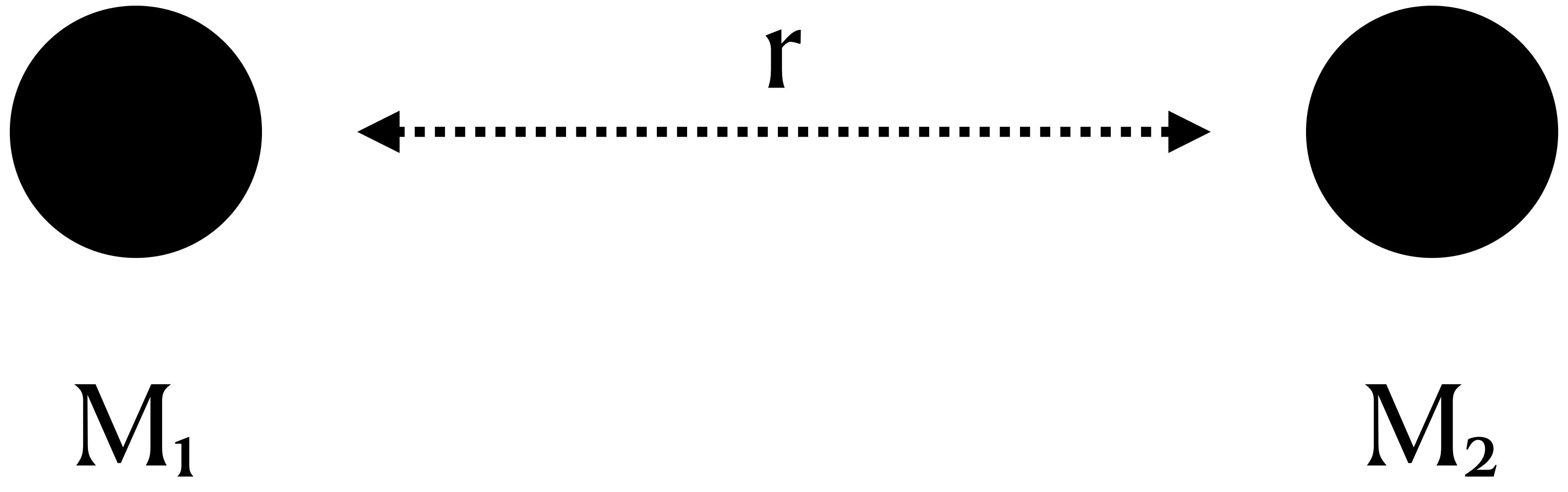

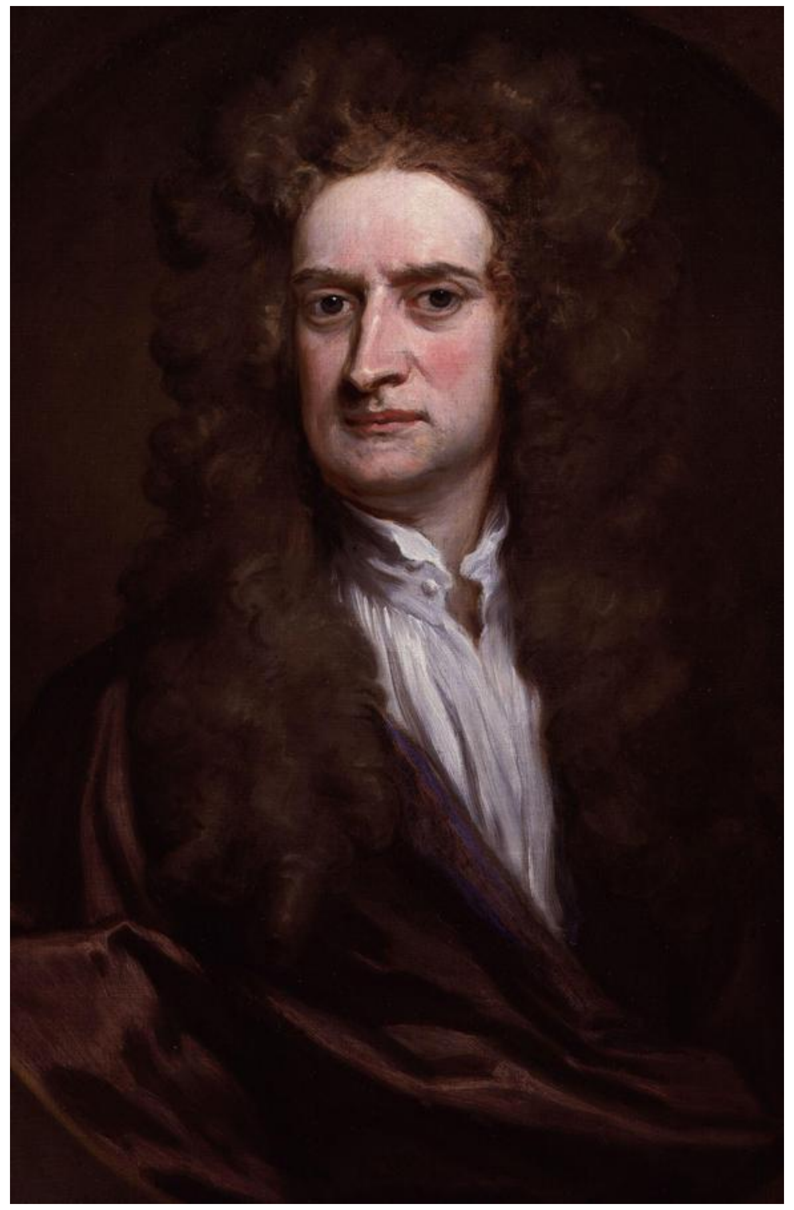

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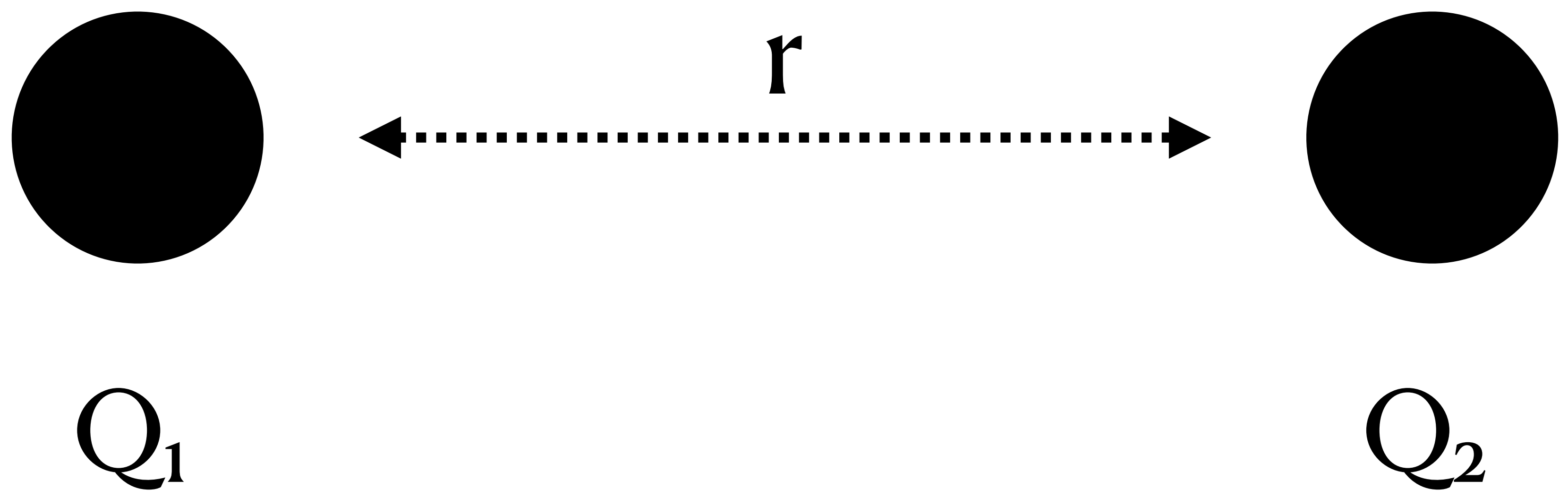
What happens at small r?

What happens at large M?

Electromagnetism



$$F_E = \frac{Q_1 Q_2}{r^2}$$

What happens at small r? Large Q?

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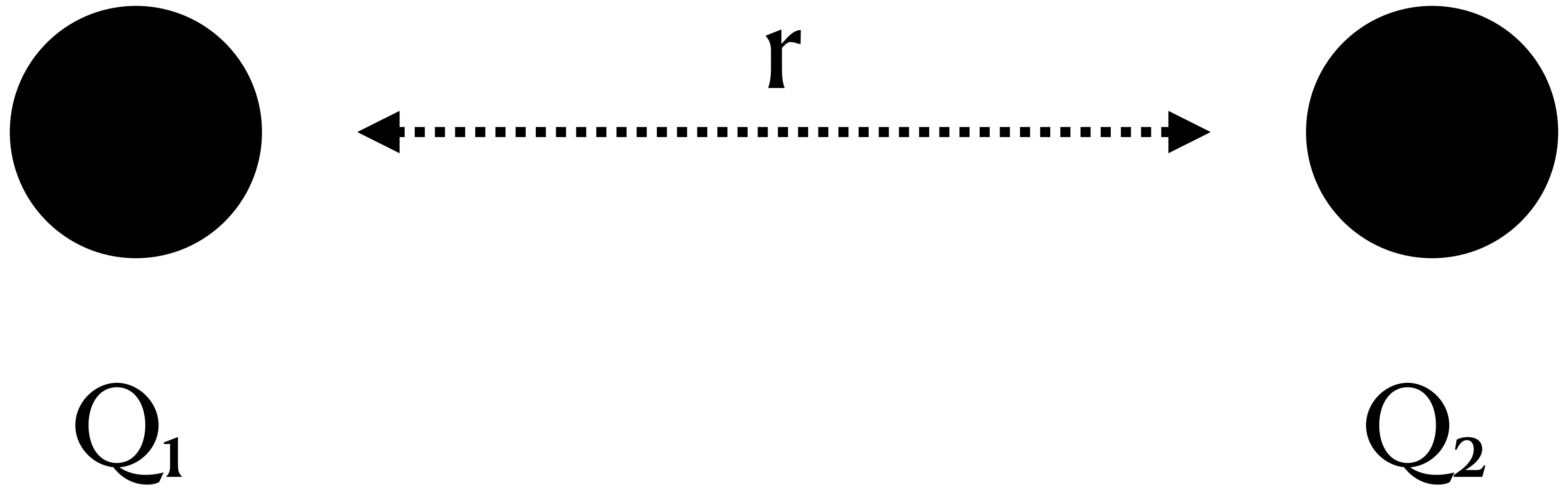

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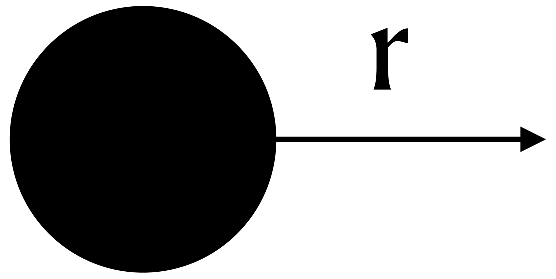

$$U = \int_{r_i}^{r_f} d^3 r \left(\frac{Q_1}{r^2}\right)^2 = \int_{r_i}^{r_f} dr \frac{Q_1^2}{r^2} = Q_1^2 \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

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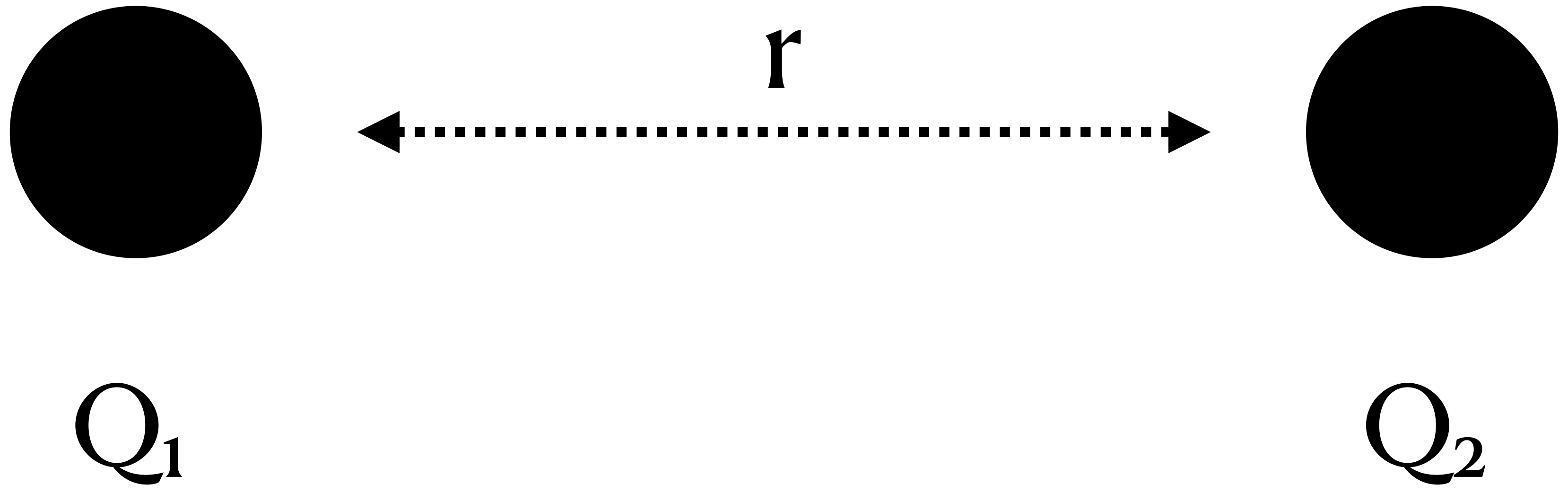

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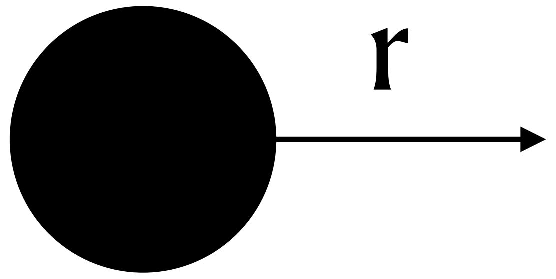

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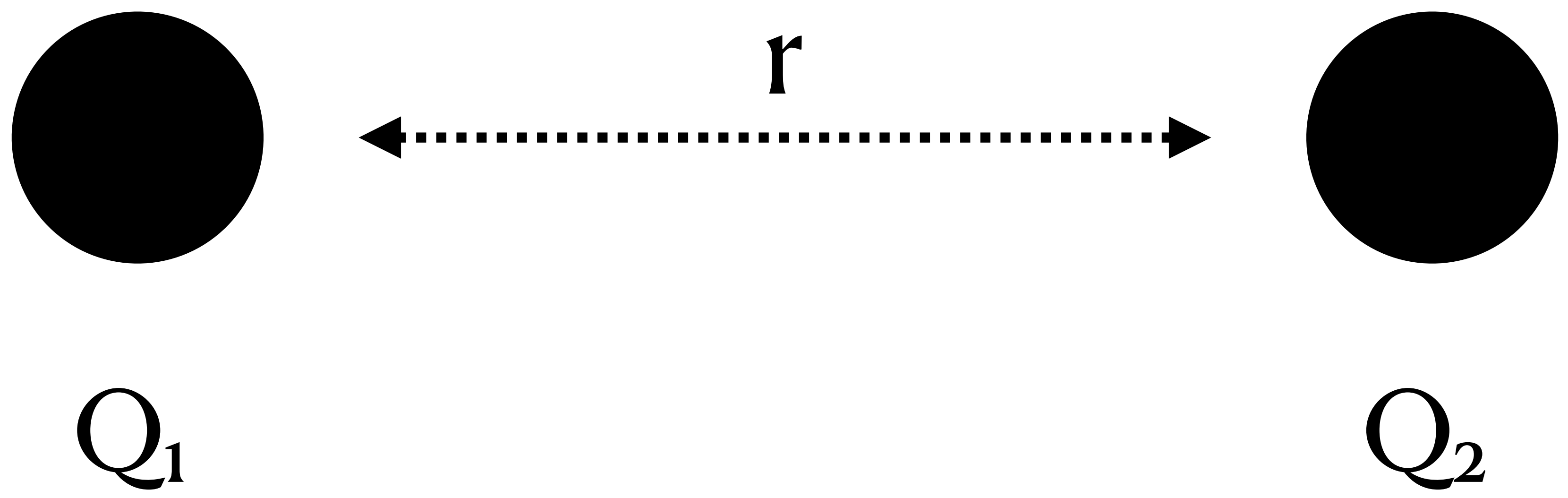
$$U \to \infty \text{ as } r_i \to 0$$

Finite Electron mass - theory wrong at short distance!

Classical theory replaced by quantum mechanics

Electromagnetism



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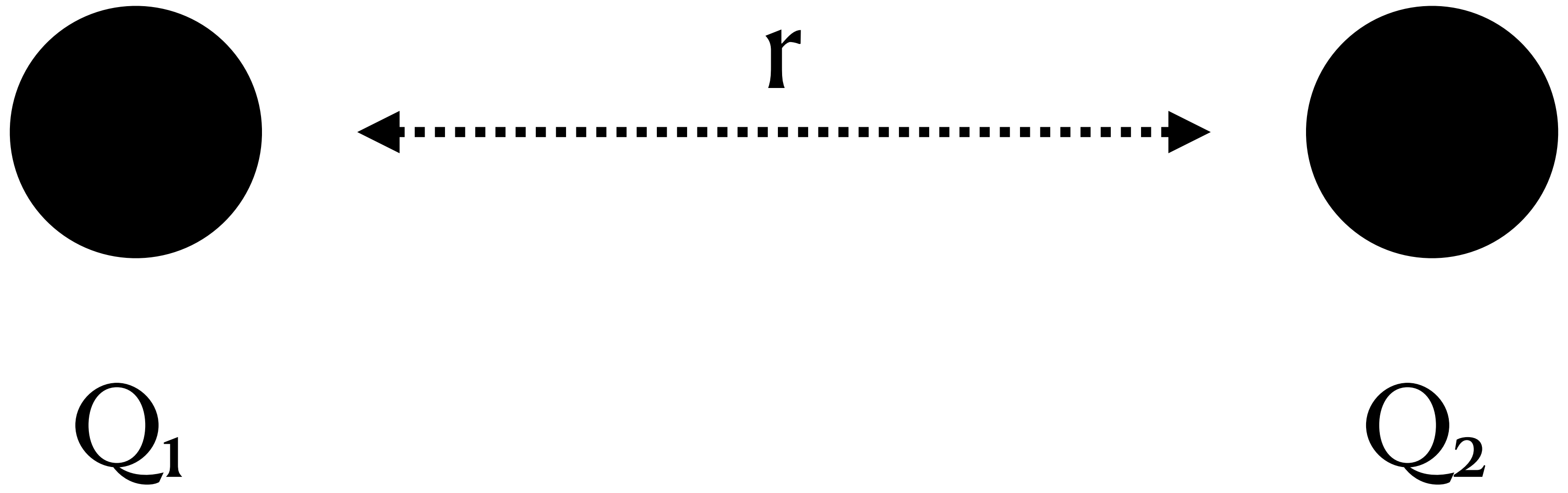

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$$U > 2m_e$$

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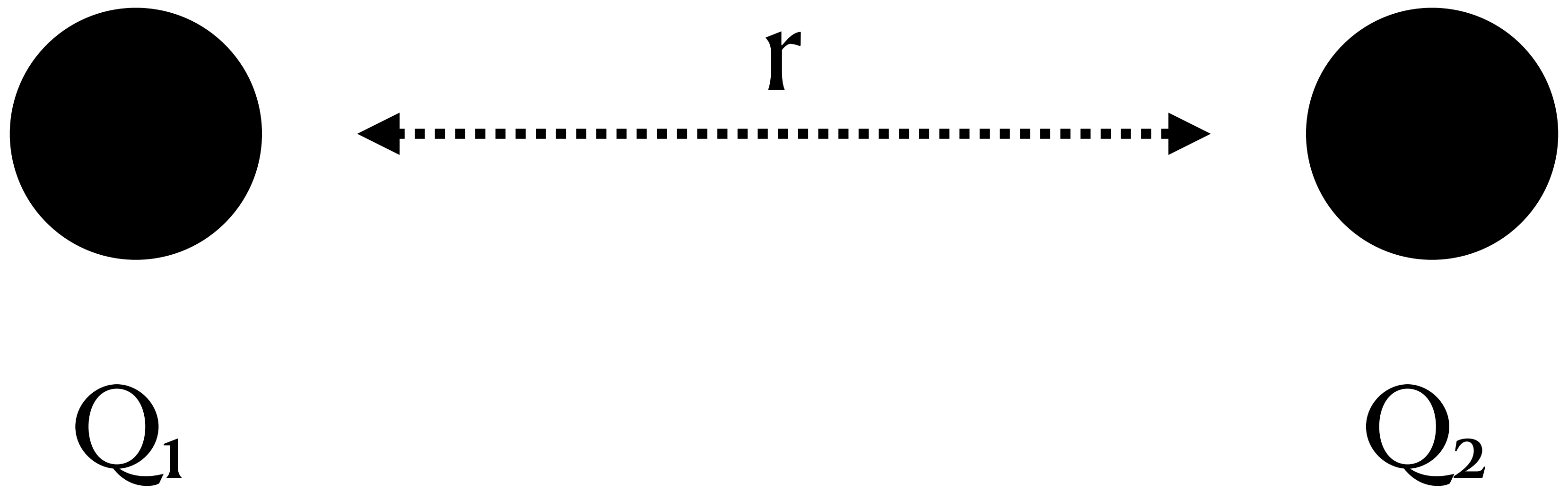

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Pair produce electrons + positrons - neutralize field

Classical theory replaced by quantum mechanics

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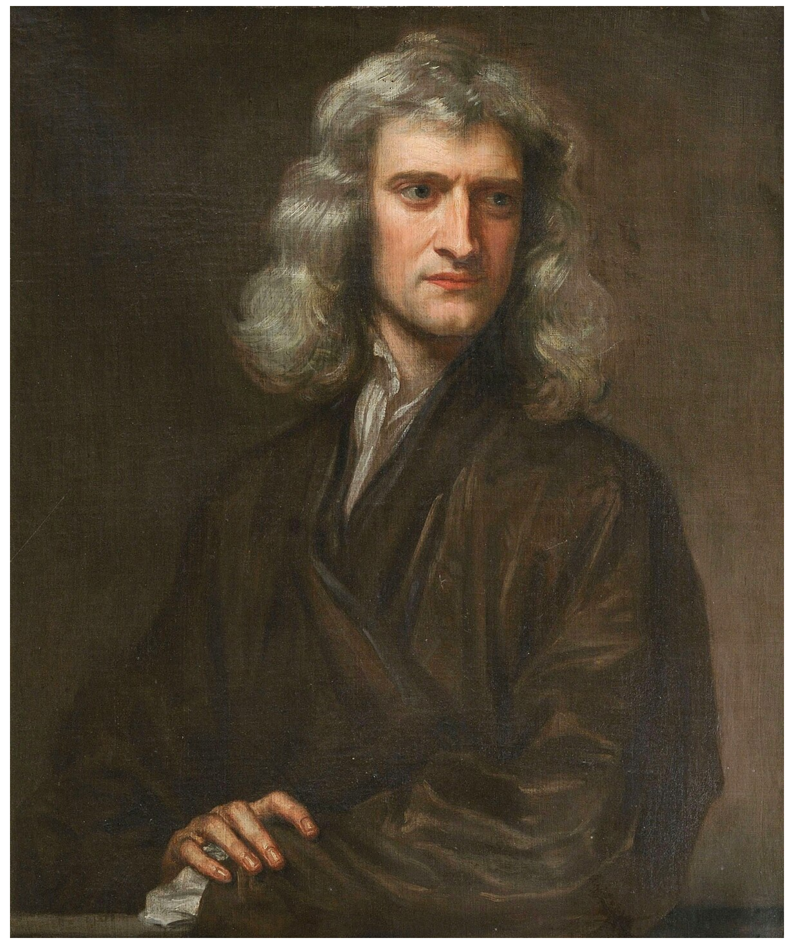

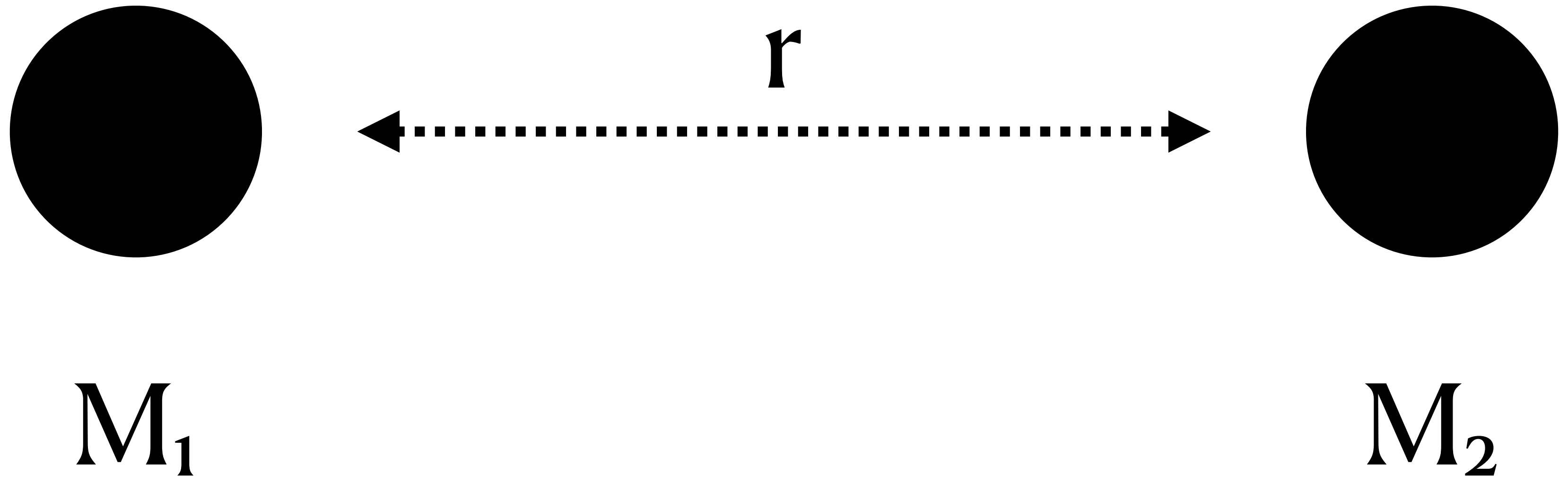

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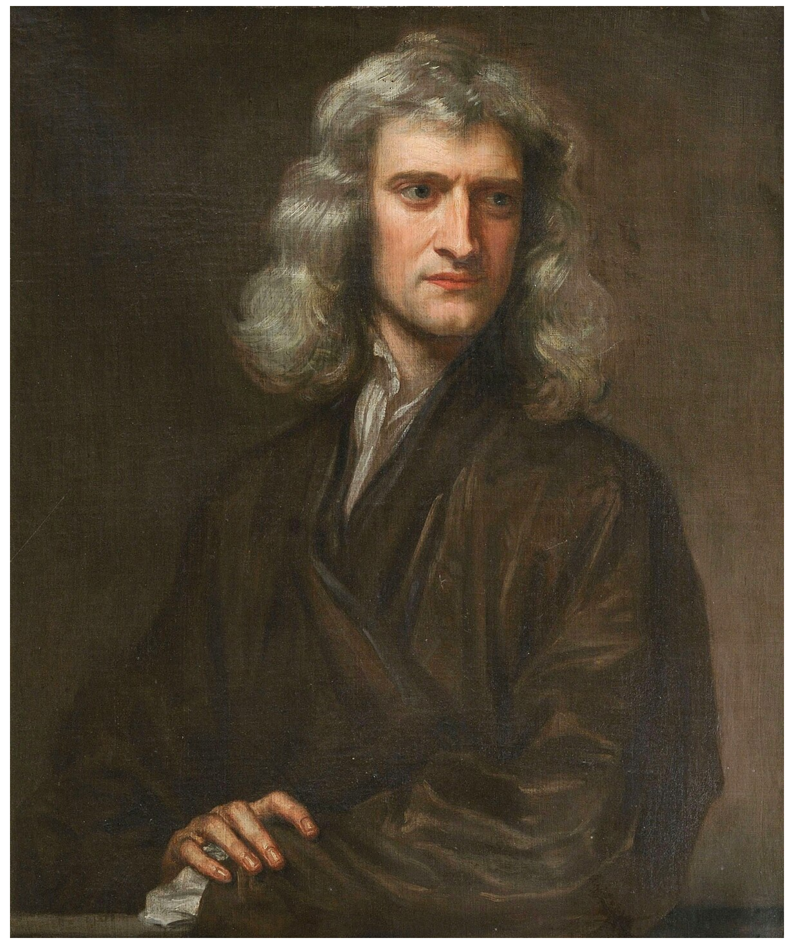

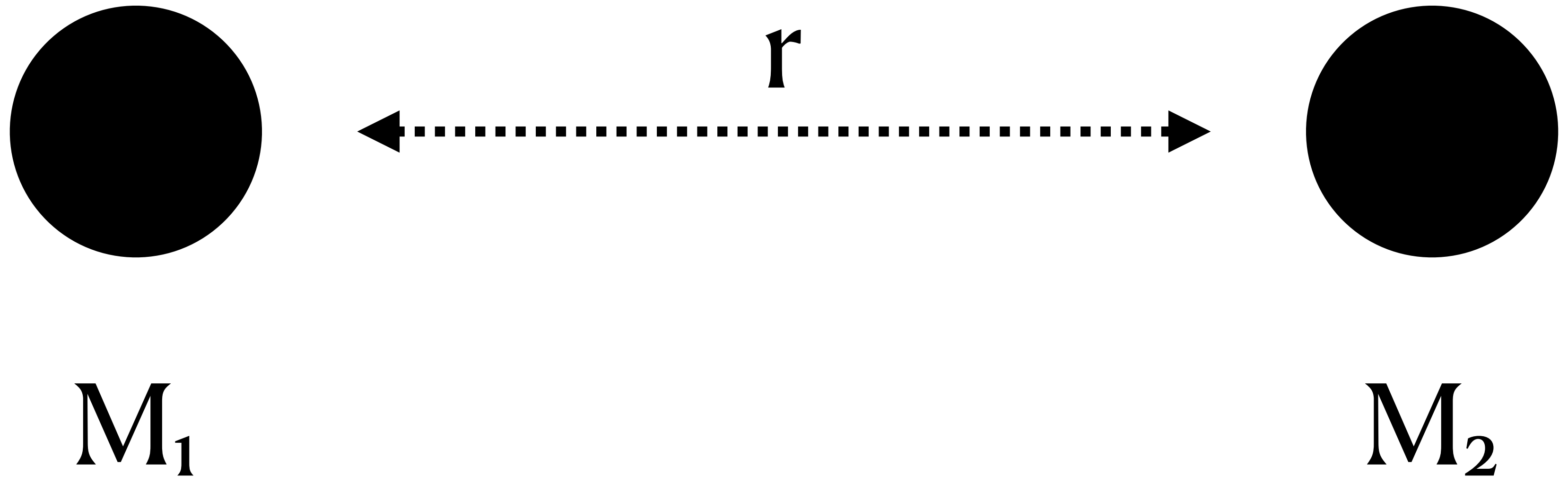

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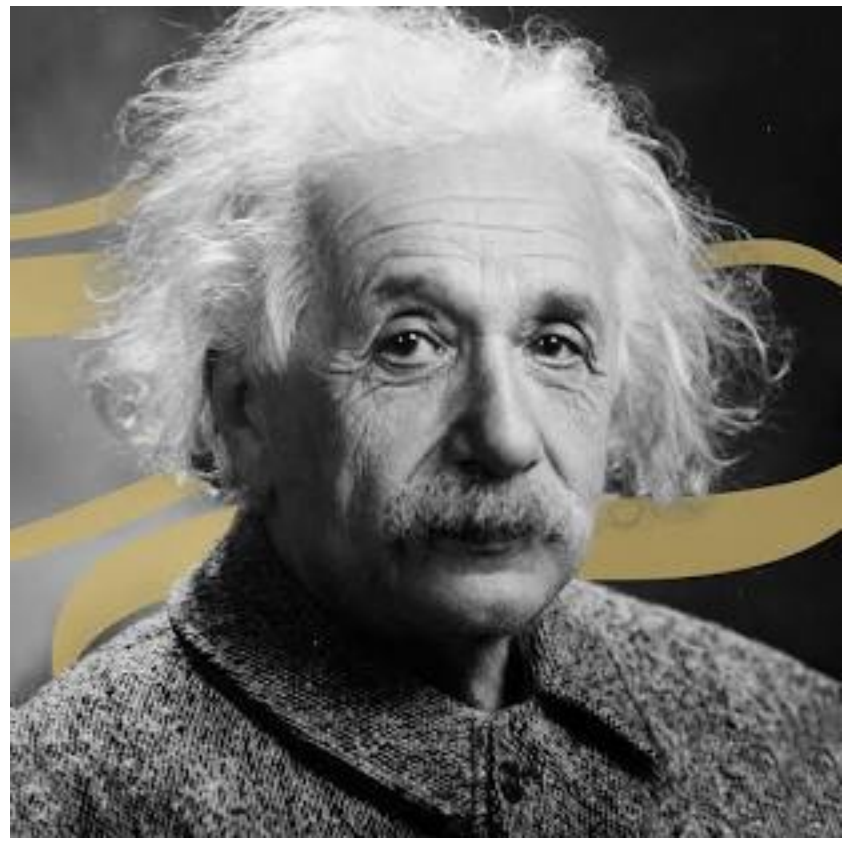

Large M

Gravity produces more gravity - interacts with itself. Non-Linear

$$G_{\mu\nu} = T_{\mu\nu}$$

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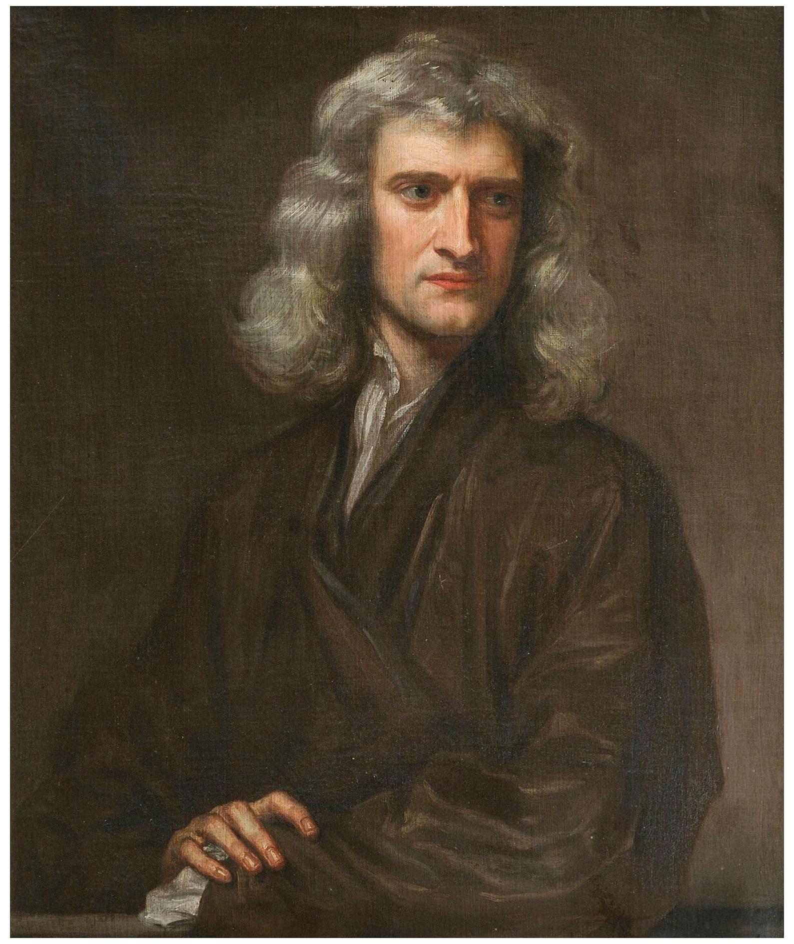

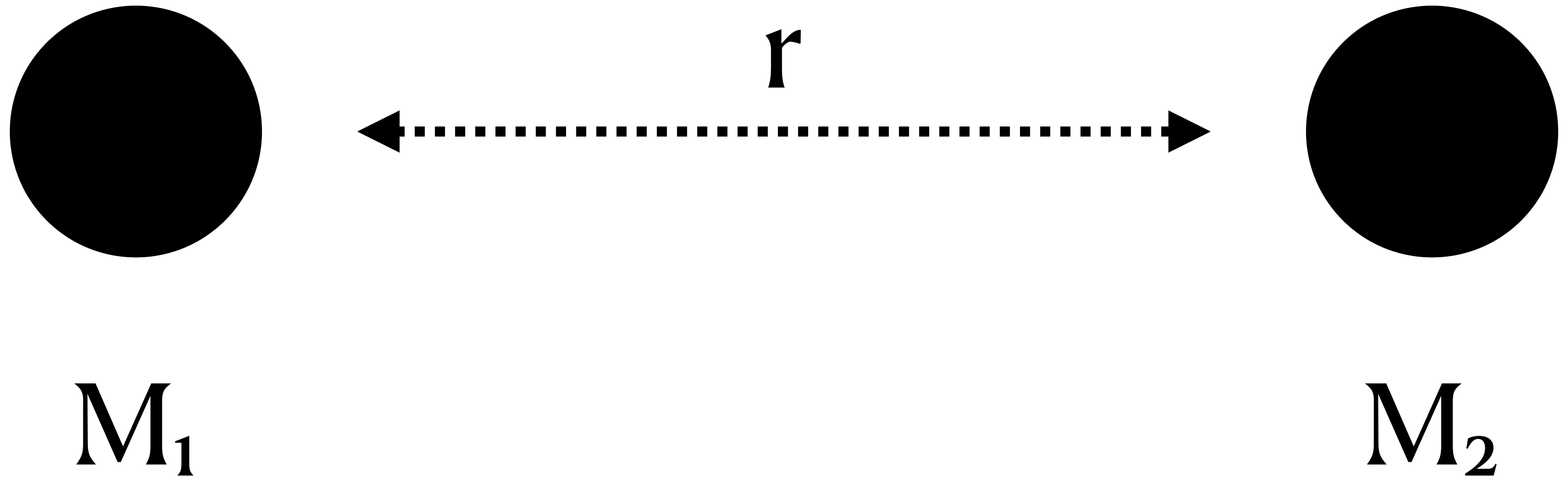

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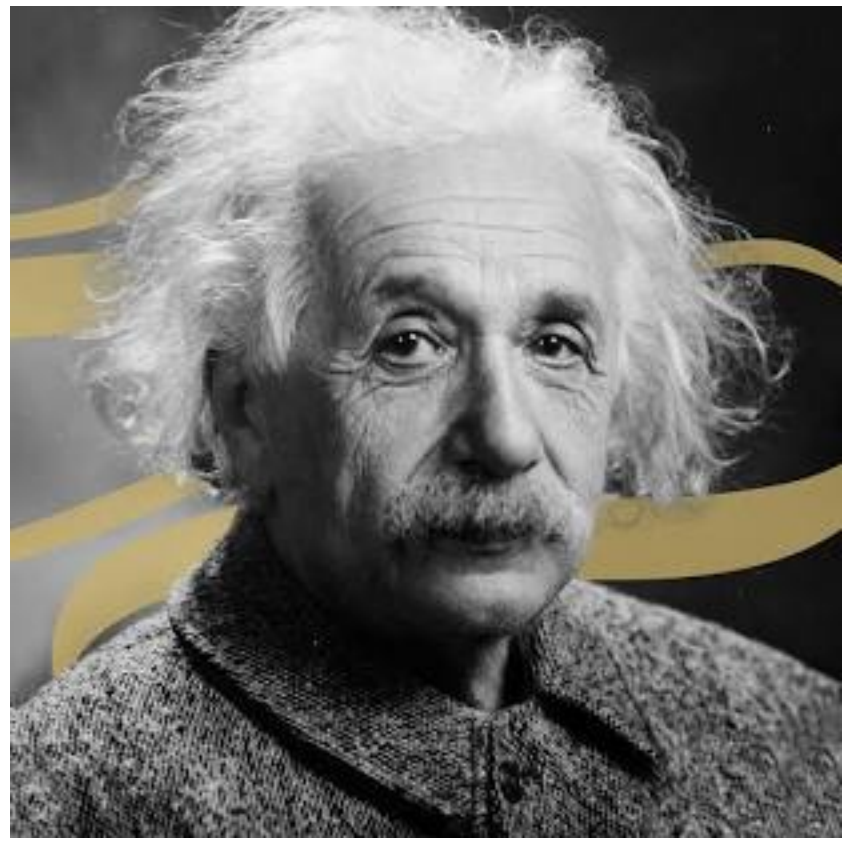

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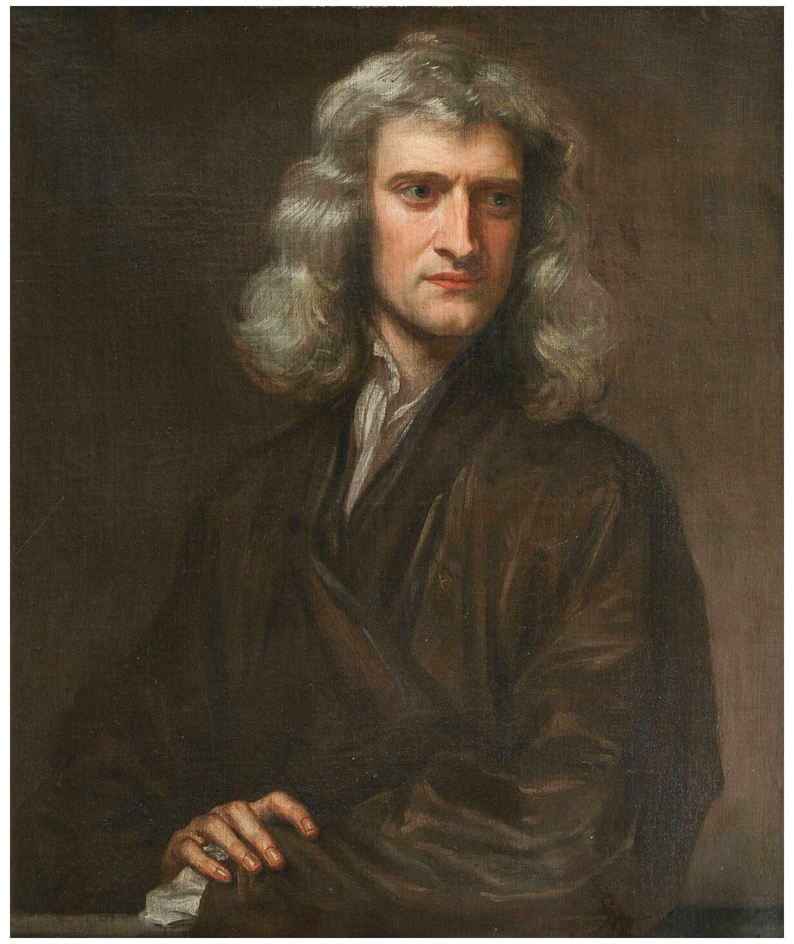
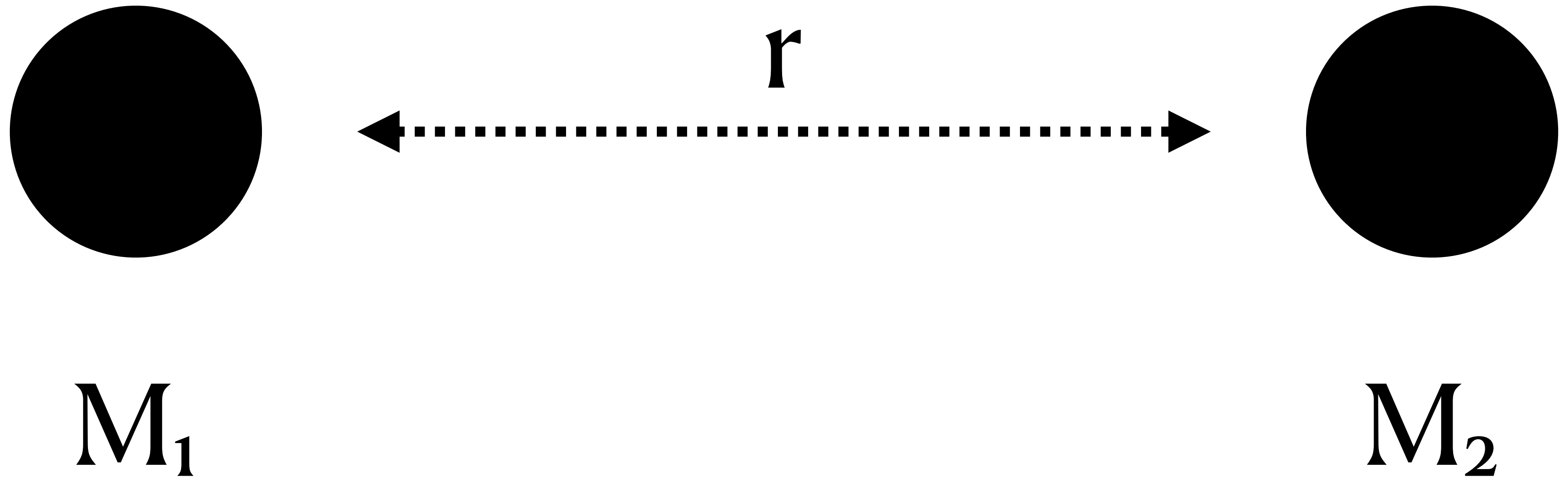
Even bigger M

No idea what happens!

Black Hole

Testing Gravity





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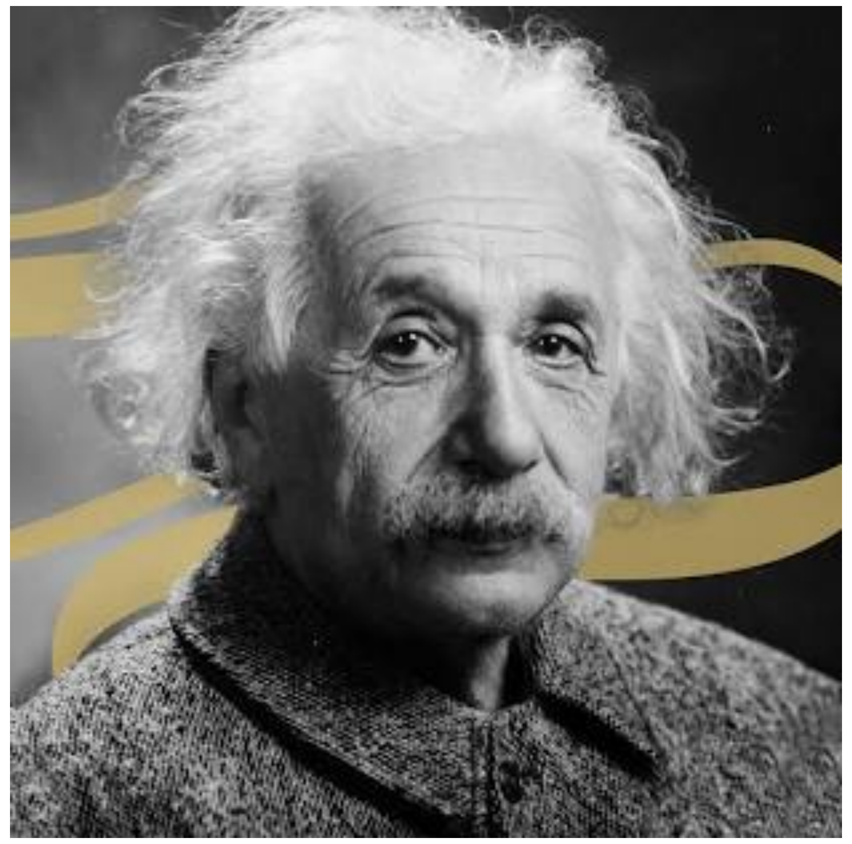

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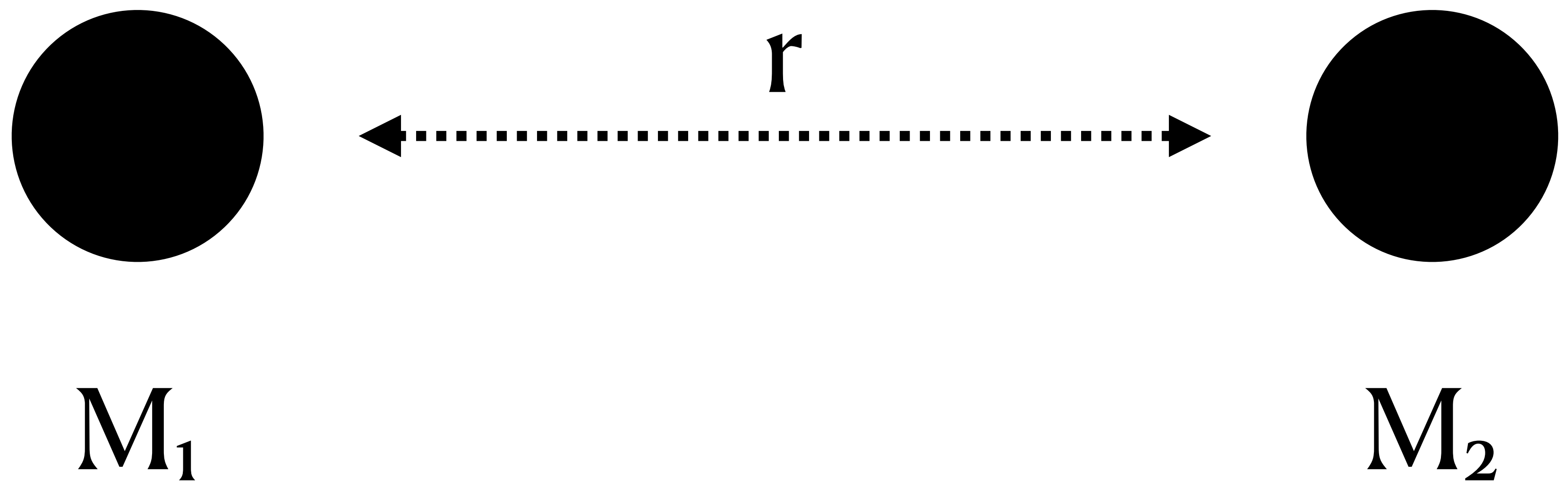

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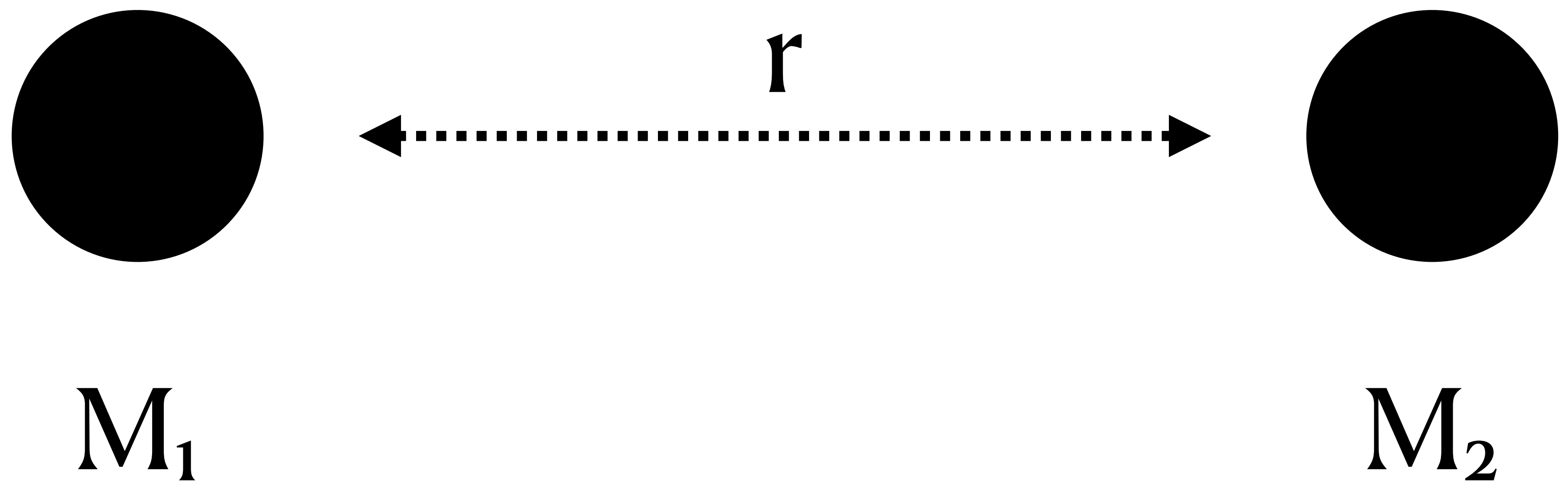

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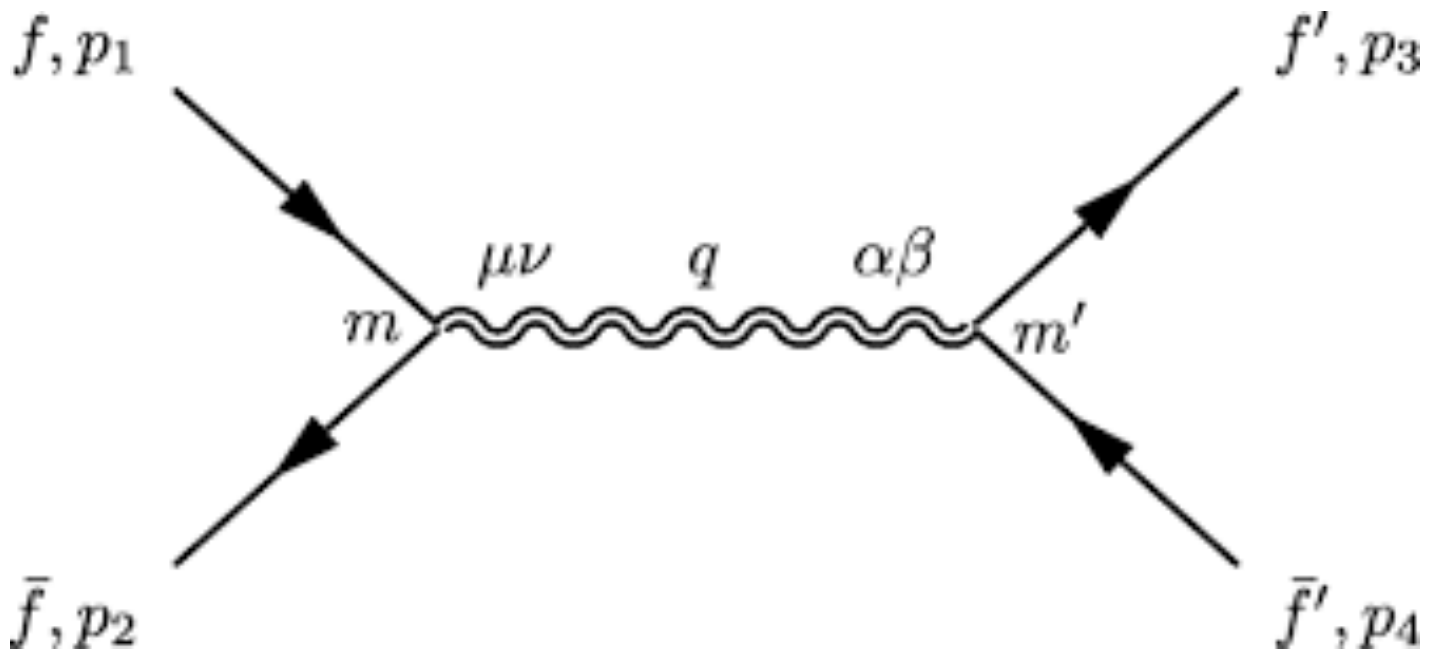
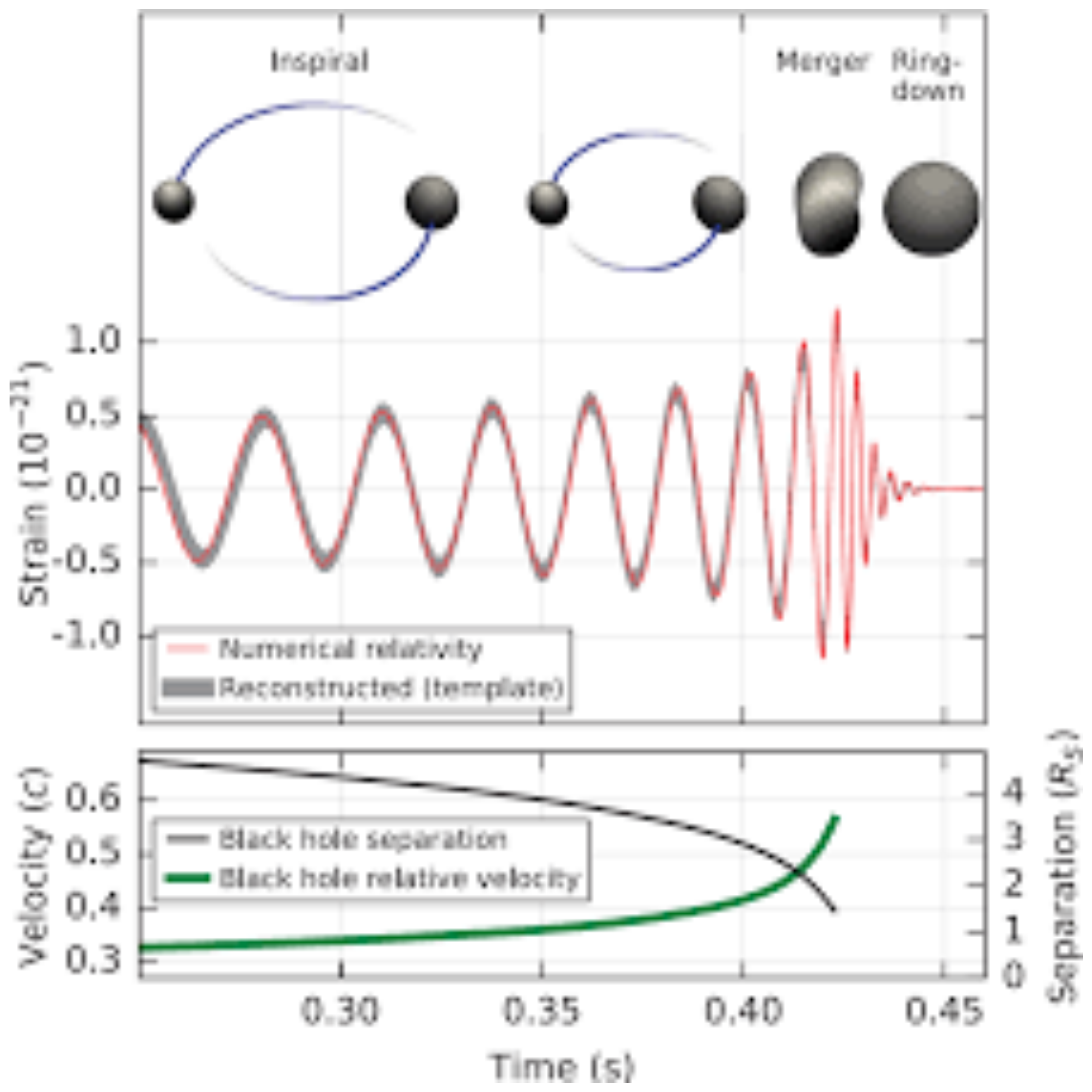


$$\propto GE^2 \to \infty \text{ as } E \to \infty$$

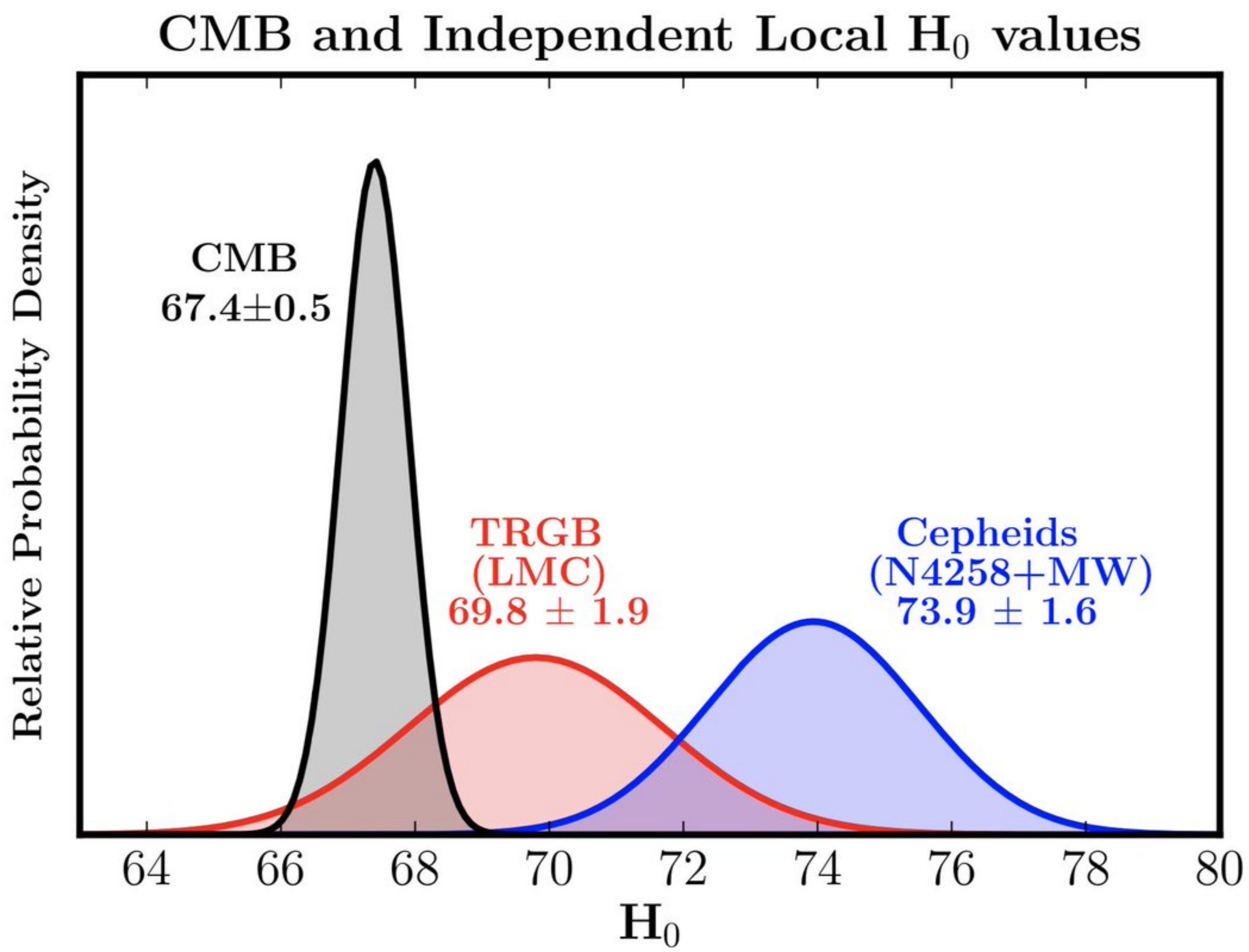
Need new theory - “quantum gravity”

Testing General Relativity

$$G_{\mu\nu} = T_{\mu\nu}$$



LIGO



Cosmology

Testing General Relativity

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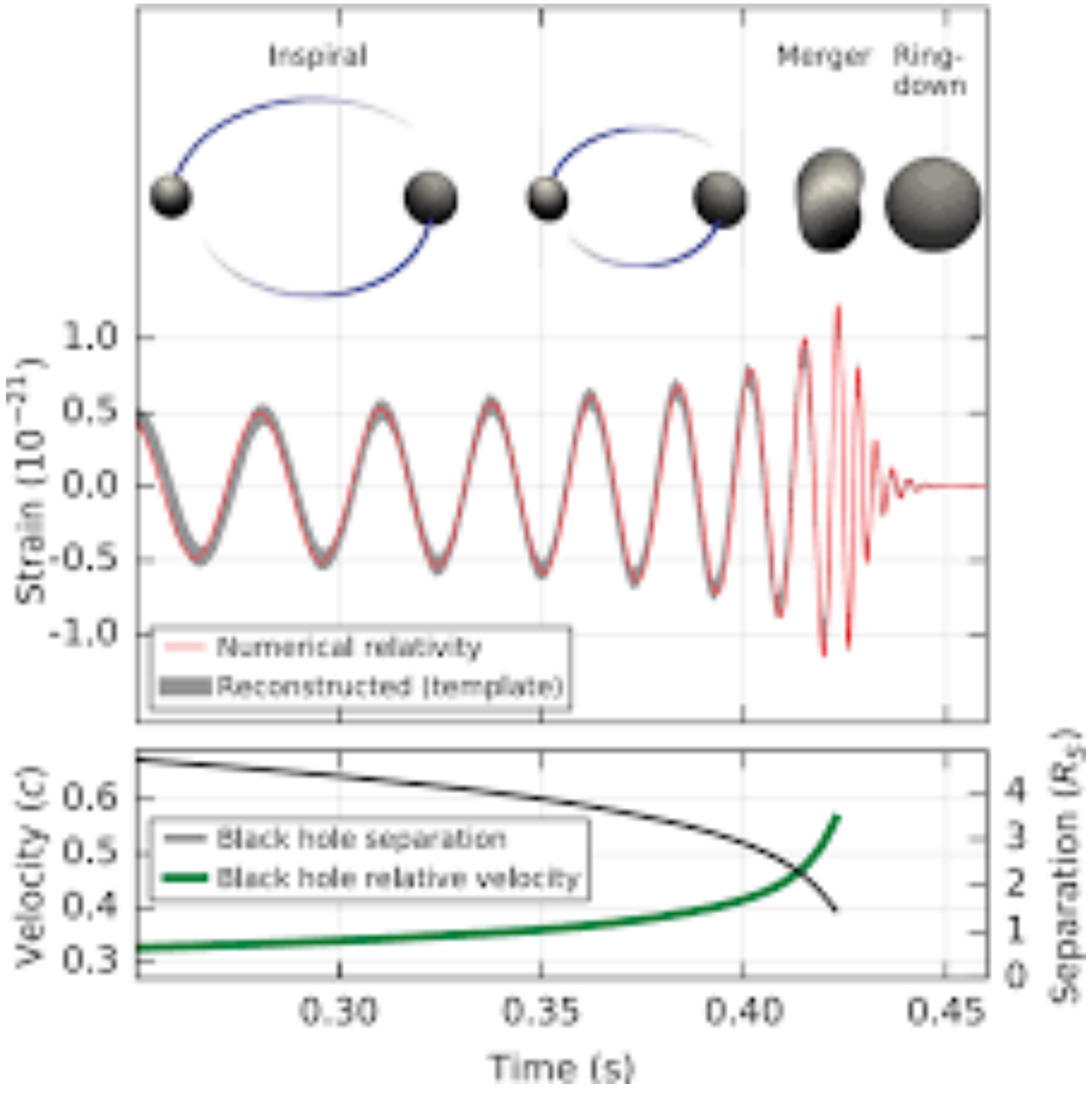

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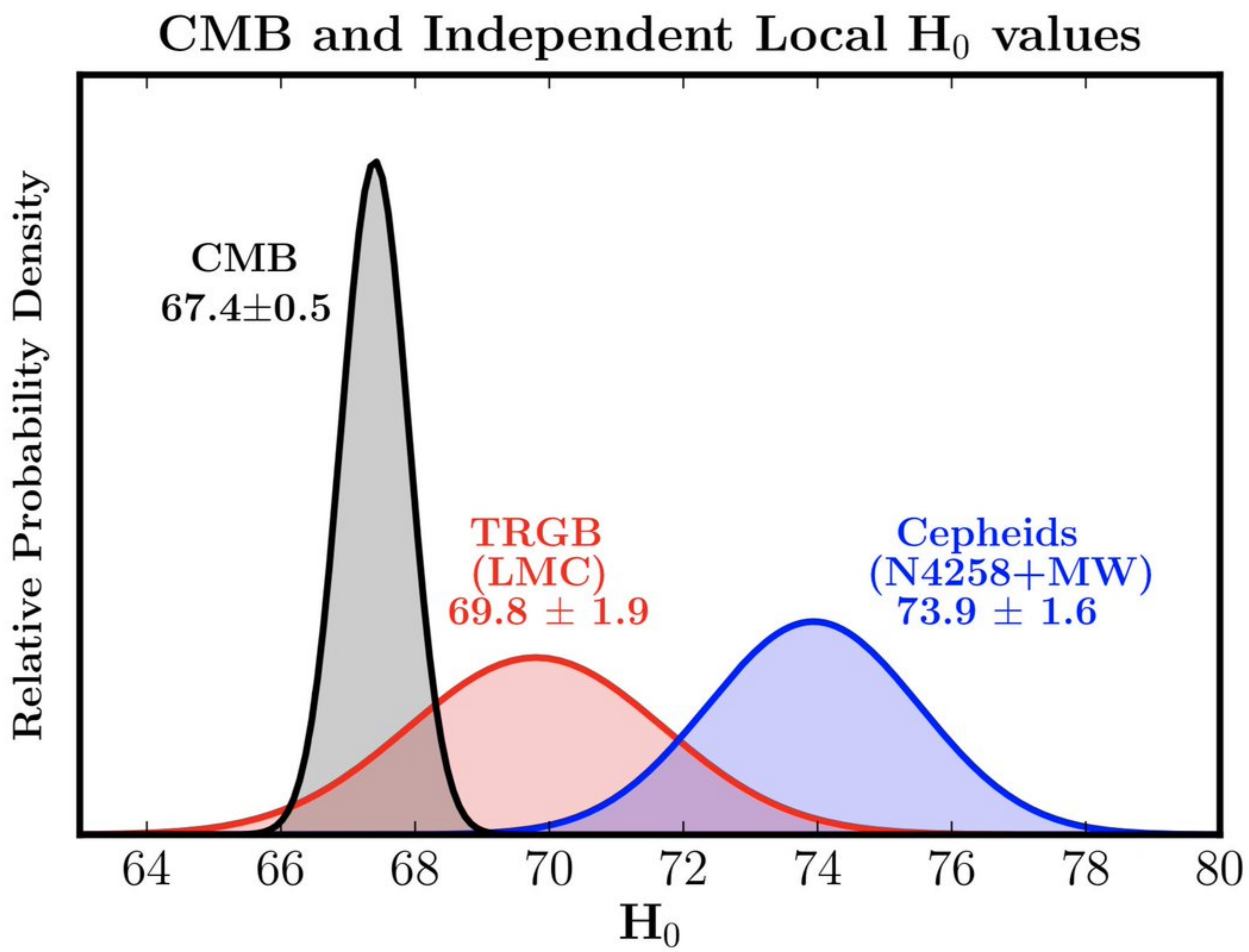
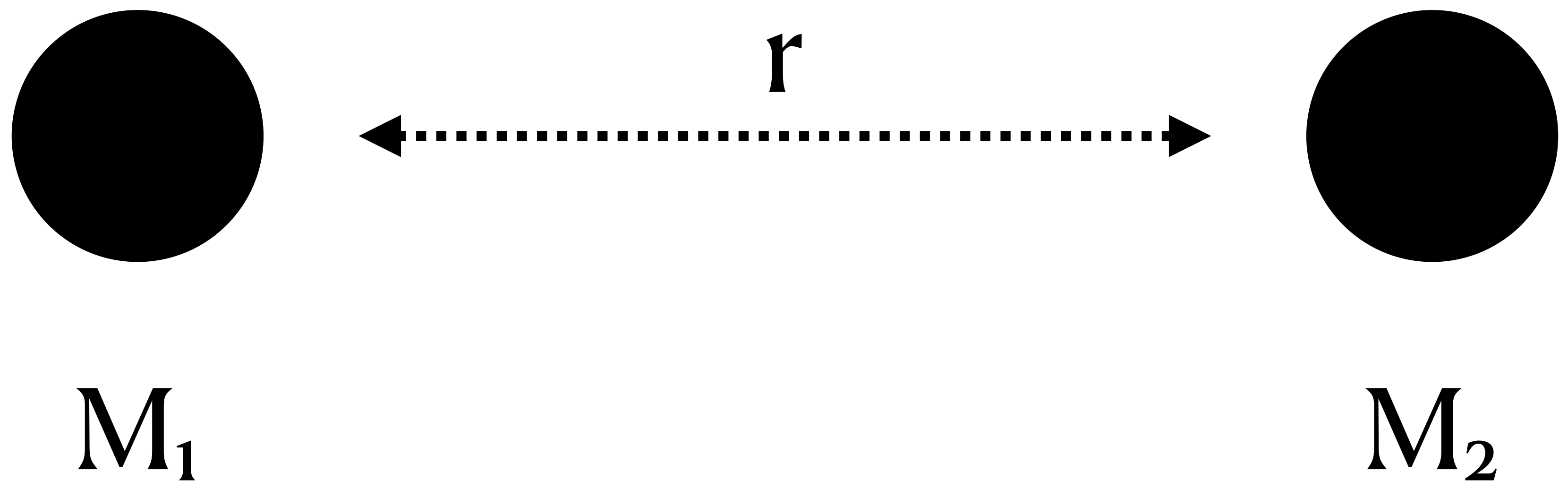


Cosmology

Tests of non-linear nature of gravity

Short Distance Tests of Gravity



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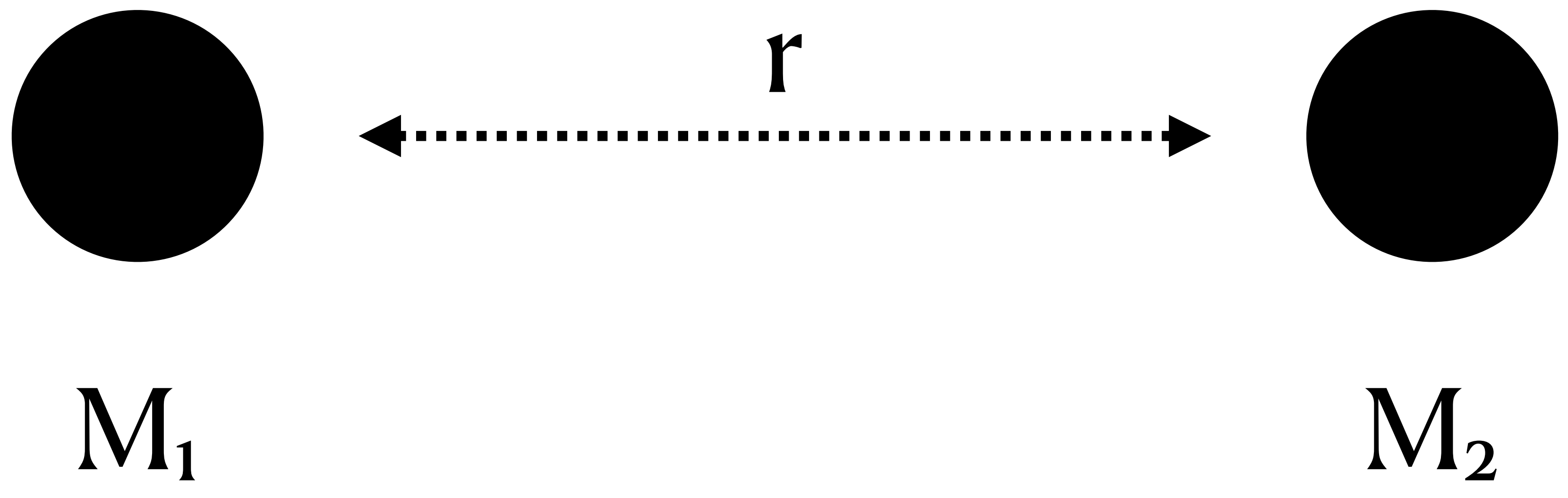

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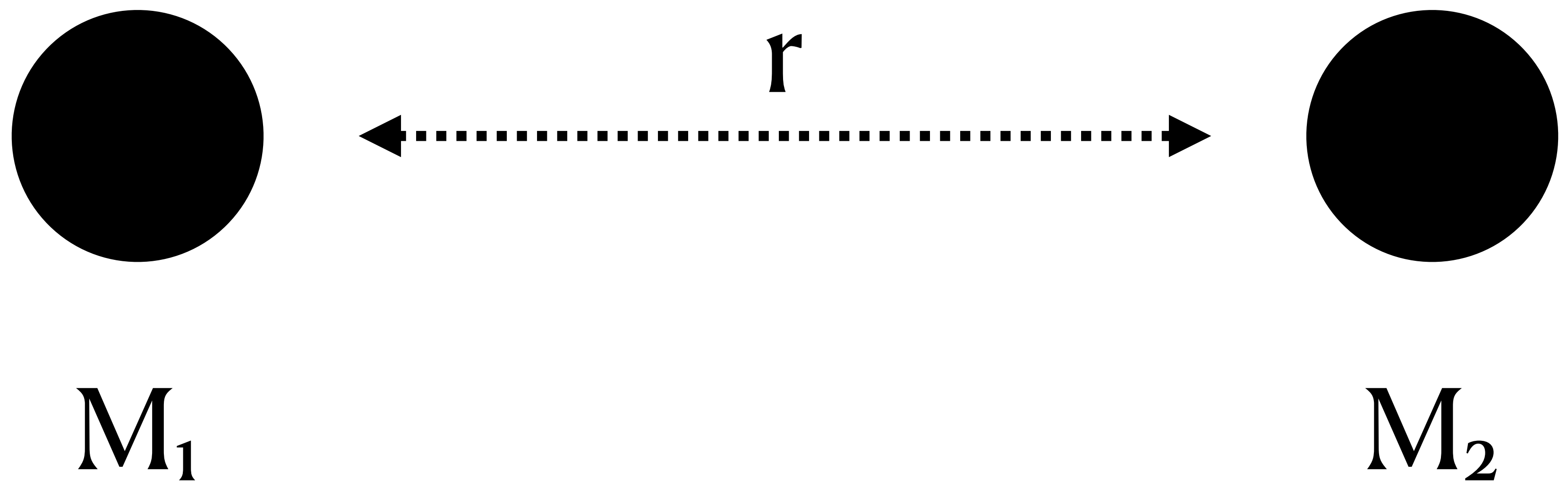

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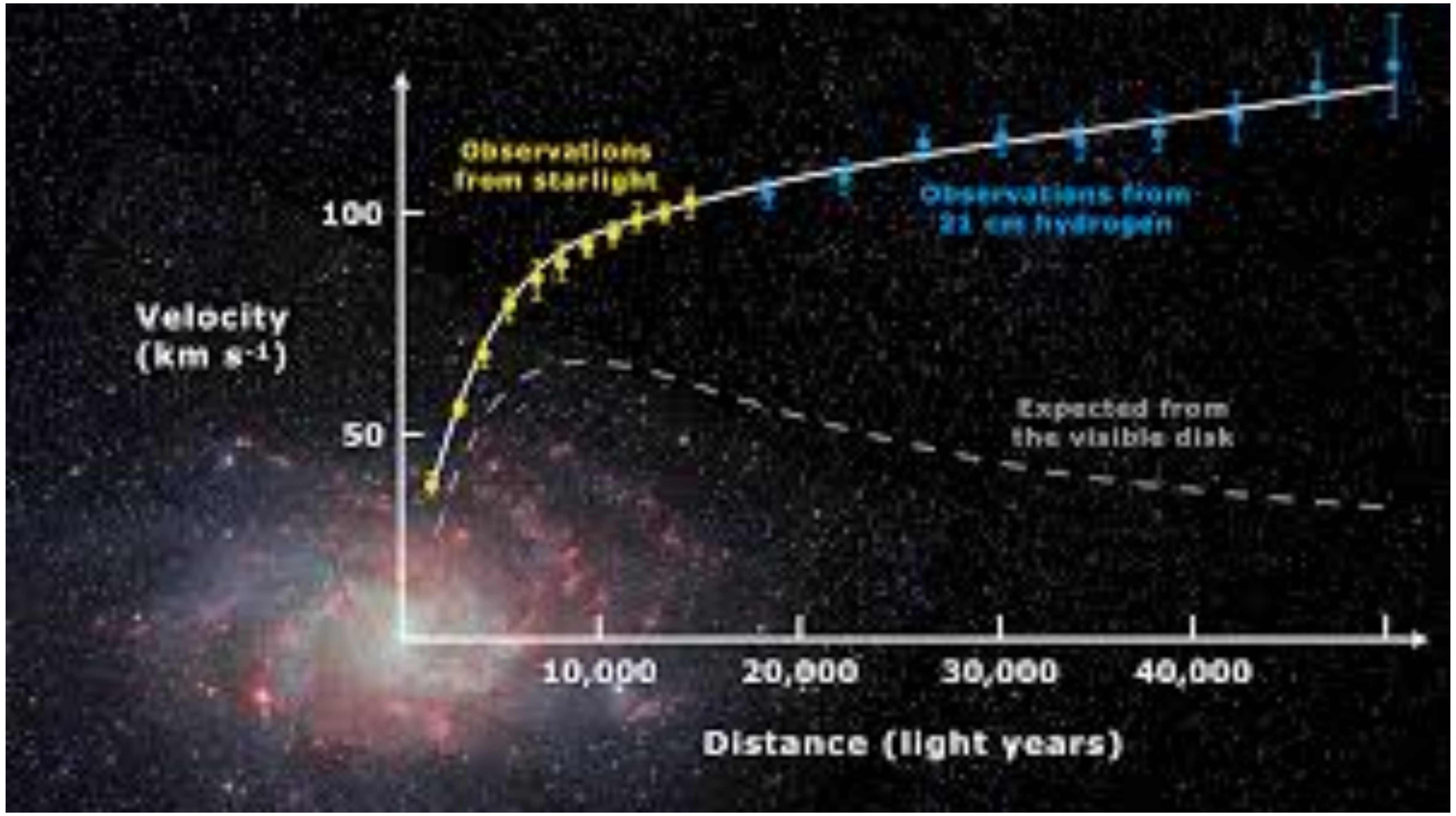
What can we find?

Testing Gravity



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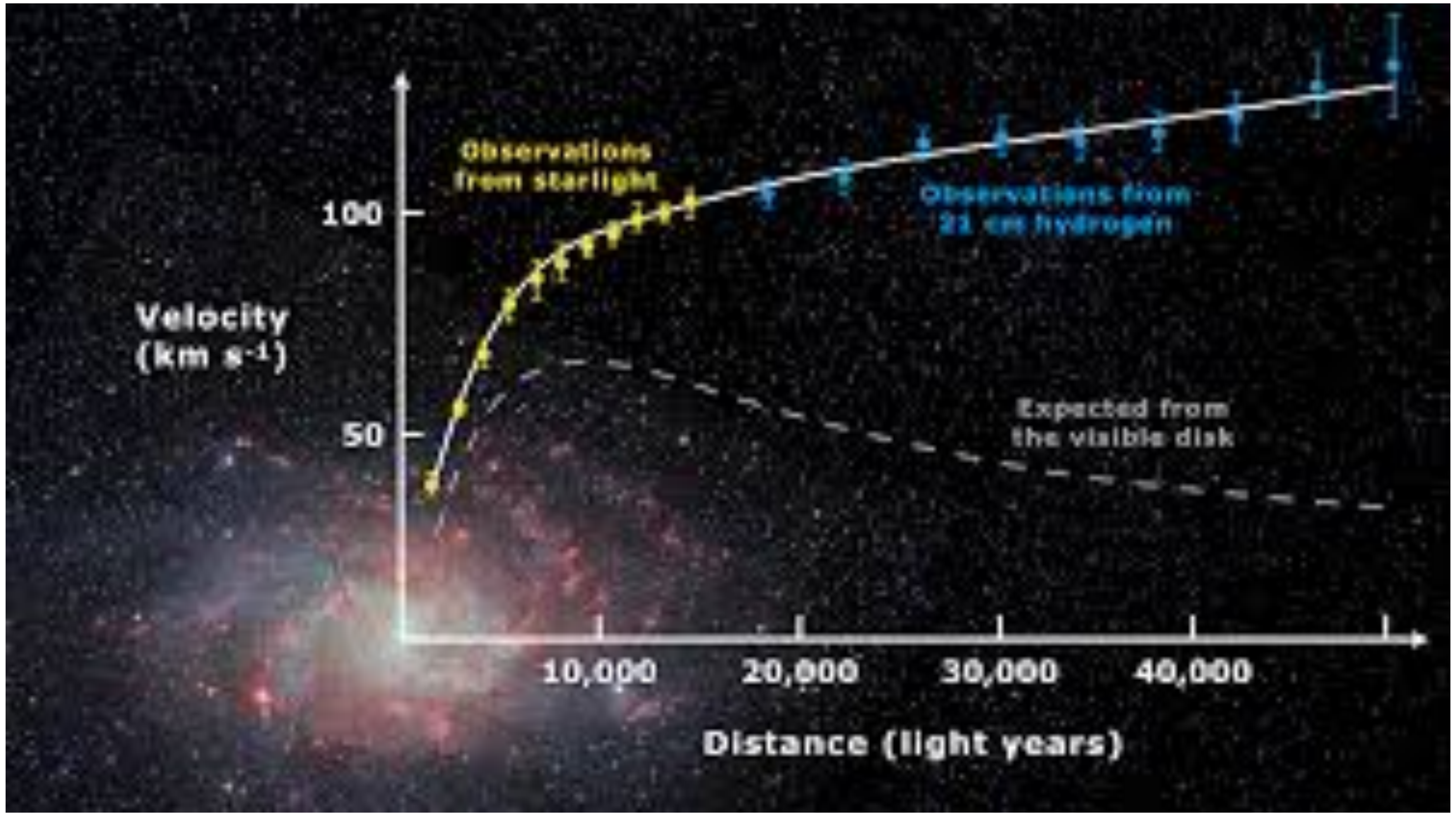
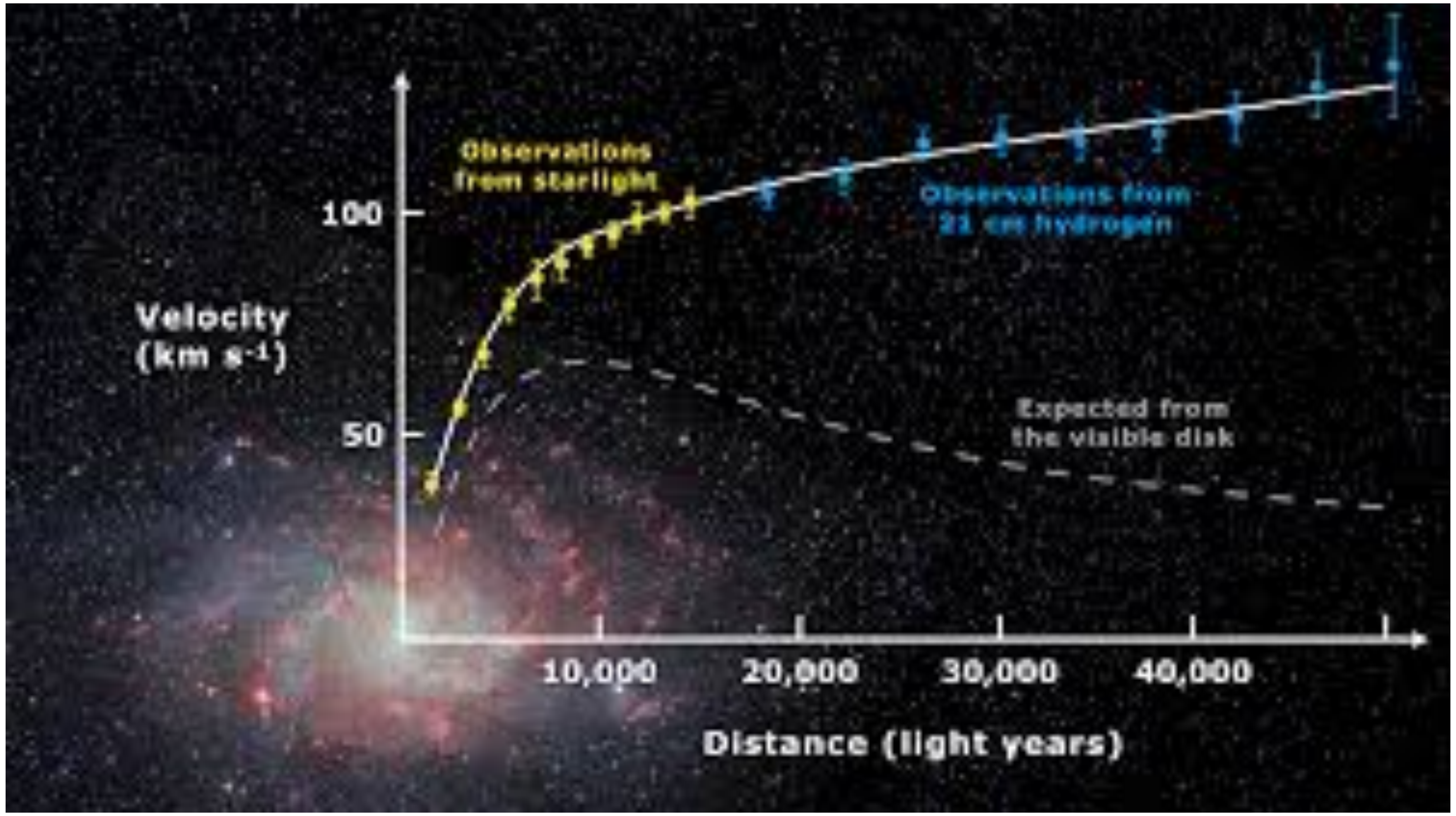


No reason to expect deviations from gravity at long distance - but found dark matter!

Testing Gravity



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What can we hope to find at short distance?

New Short Distance Forces

Light bosonic particles motivated by BSM Physics
(e.g. radions, moduli, relaxions)

$$\mathcal{L} \supset y_q \phi \bar{q} q + \frac{\phi}{f_\gamma} F_{\mu\nu}^2 + \frac{\phi}{f_g} G_{\mu\nu}^2 + \frac{\tilde{h}_{\mu\nu}}{f_T} F^{\mu}{}_{\sigma} F^{\nu\sigma} + g\phi h^2 + \frac{m_\phi^2}{2}\phi^2$$

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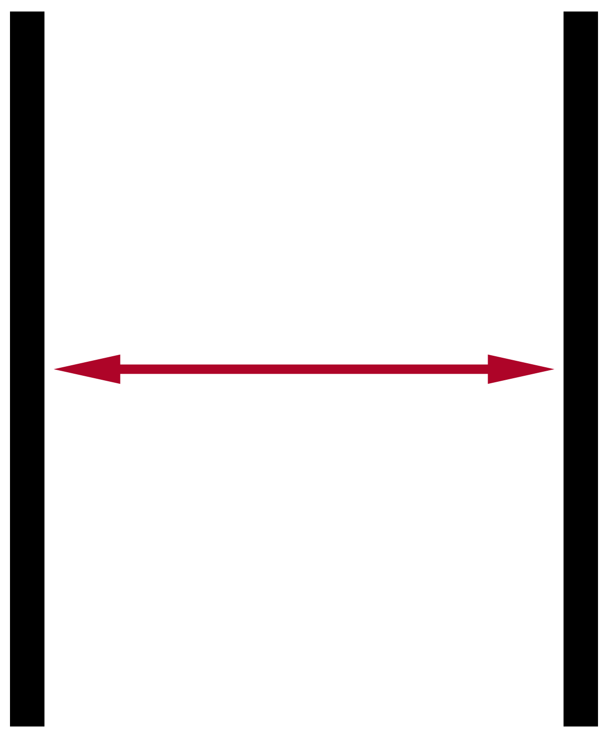
How do we find them?

Take two objects, measure anomalous forces between them



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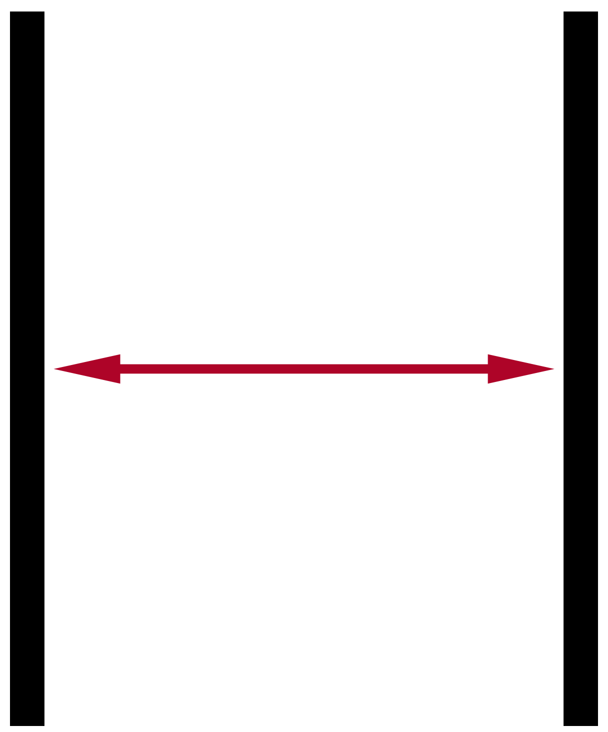

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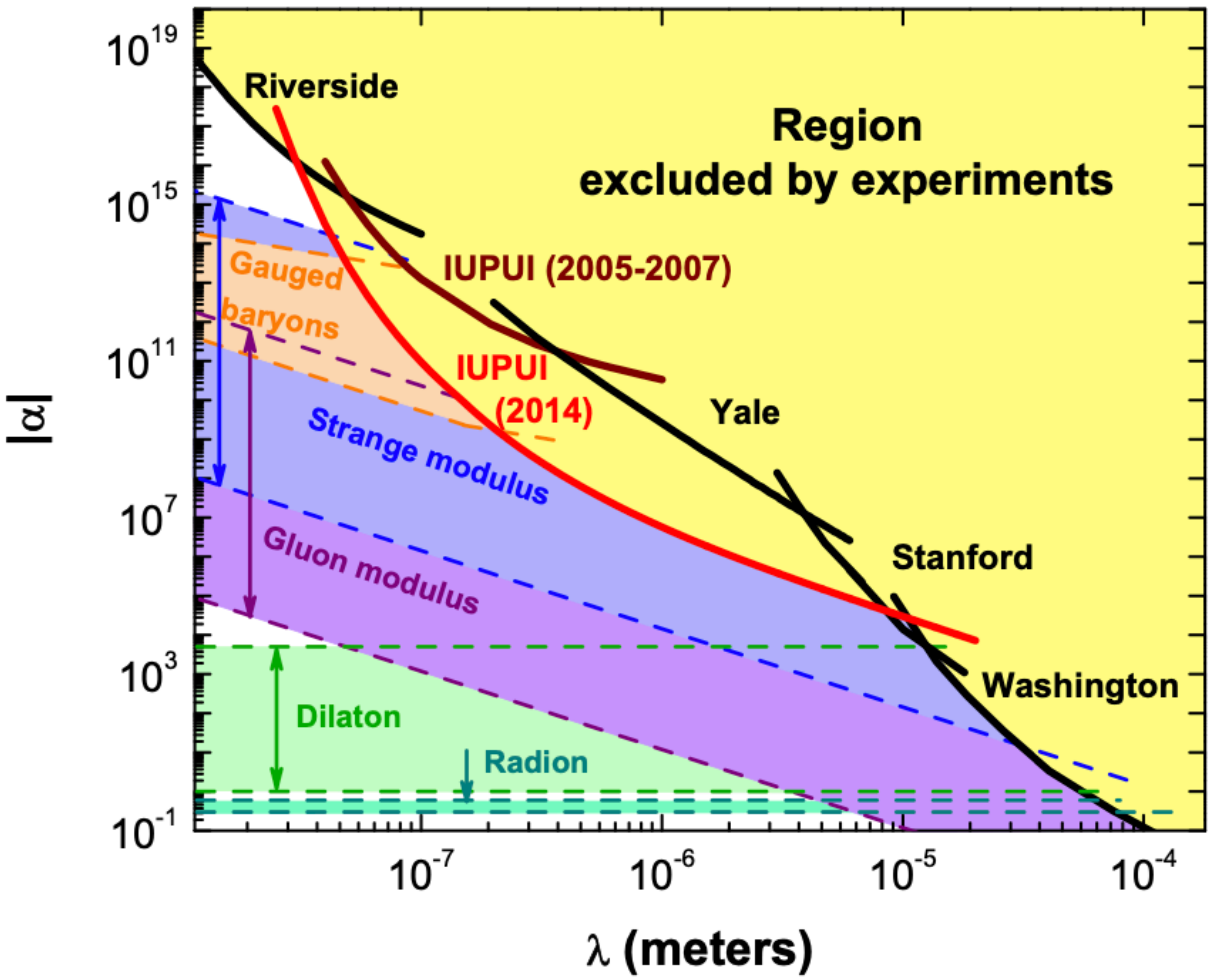
Measure Relative Acceleration

Where are we?



Very strong constraints at long (> μm) distances

Sensitivity rapidly drops at short (< μm) distances

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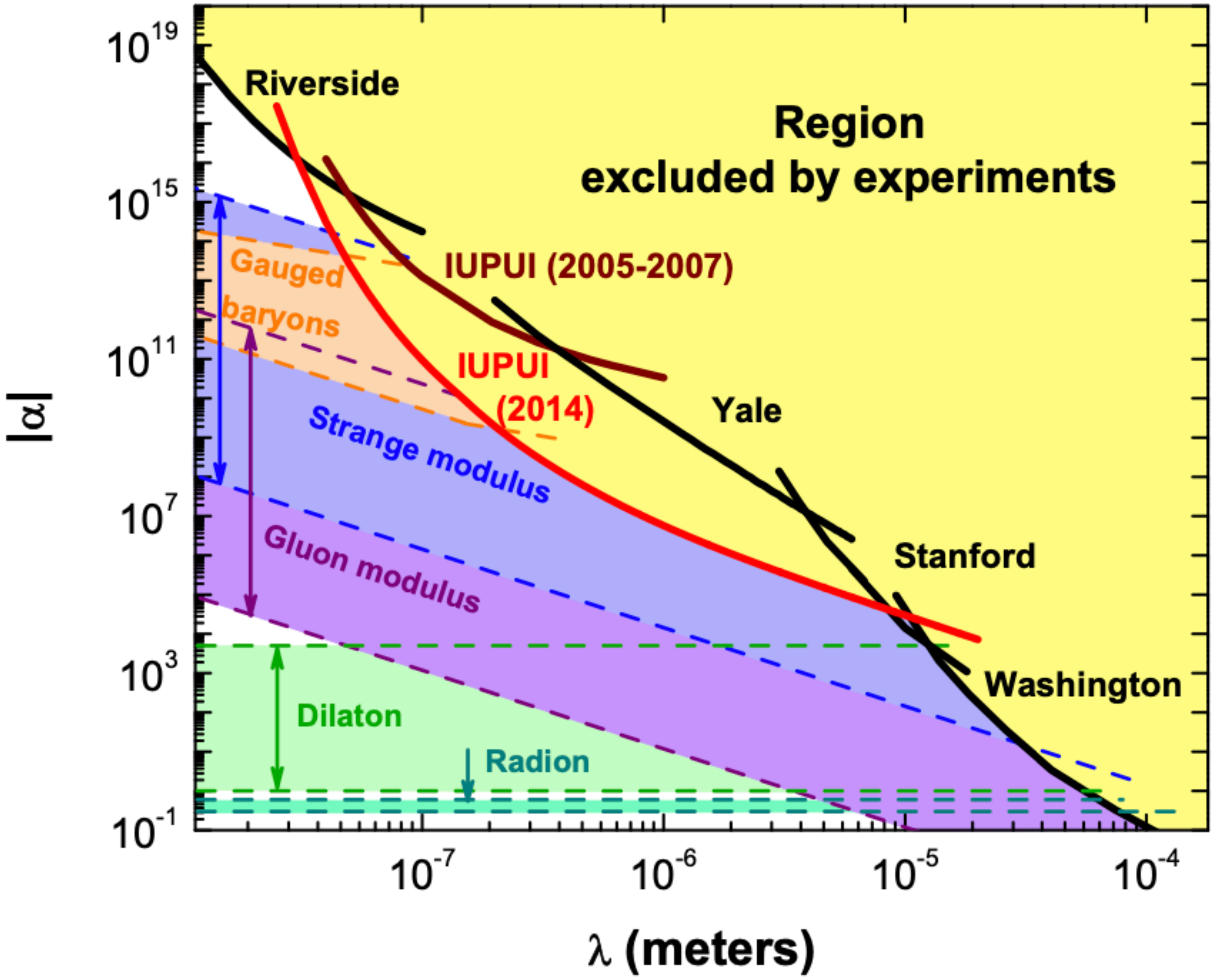

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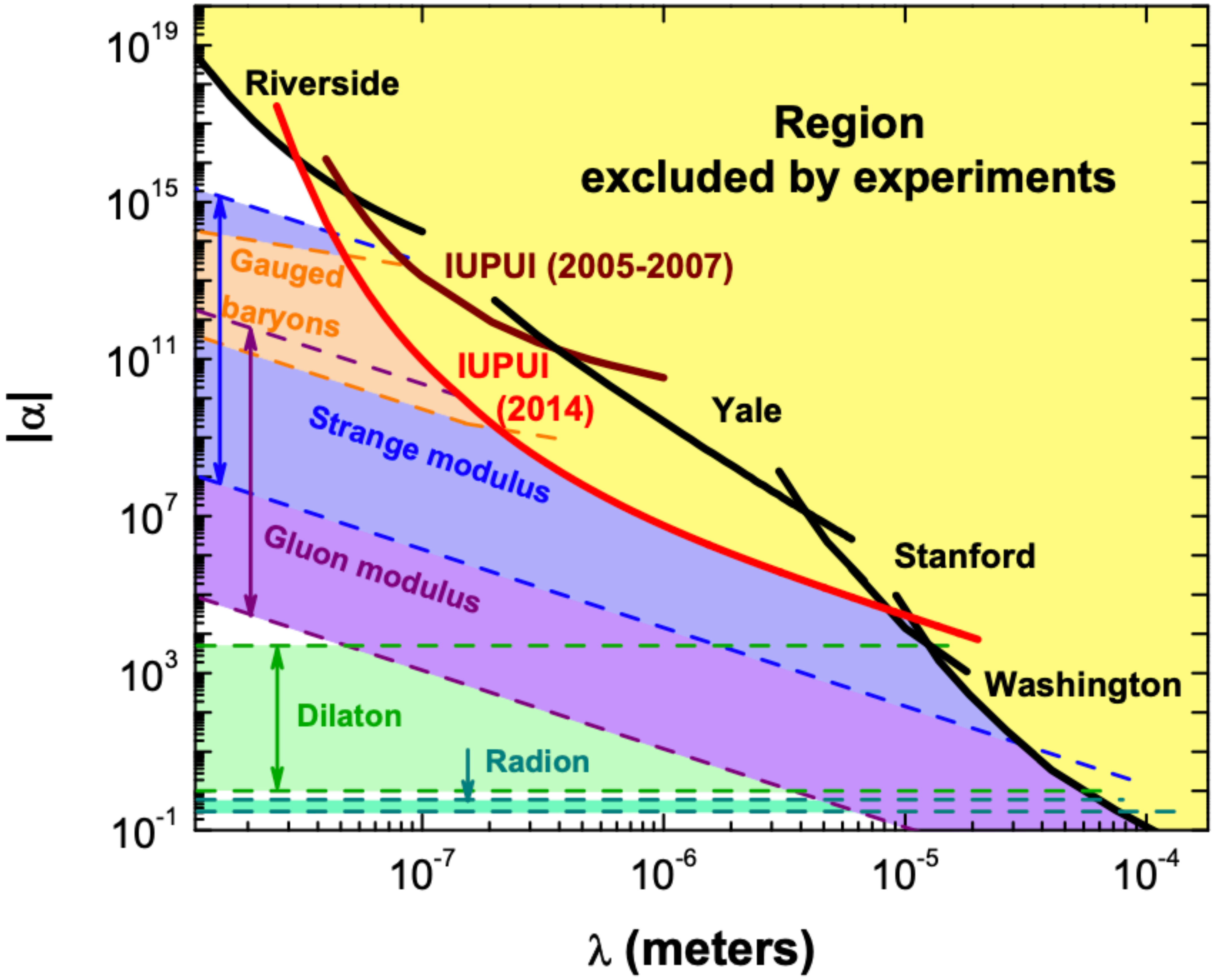

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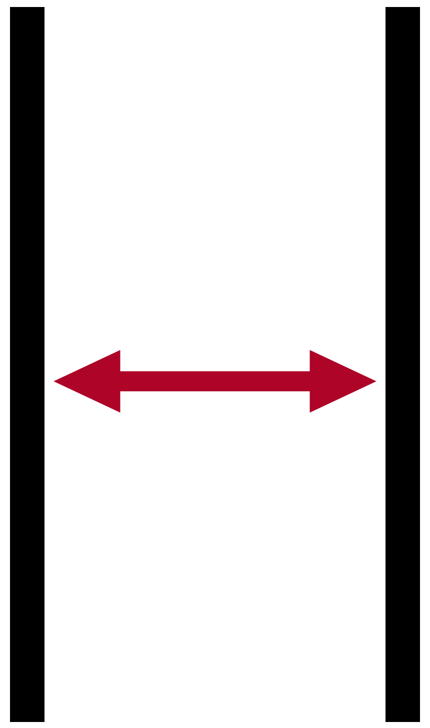

Short Range => Objects need to be close

Electromagnetism >> New Physics

Short Range => Only material within λ affected

Need to deal with thin objects with high precision

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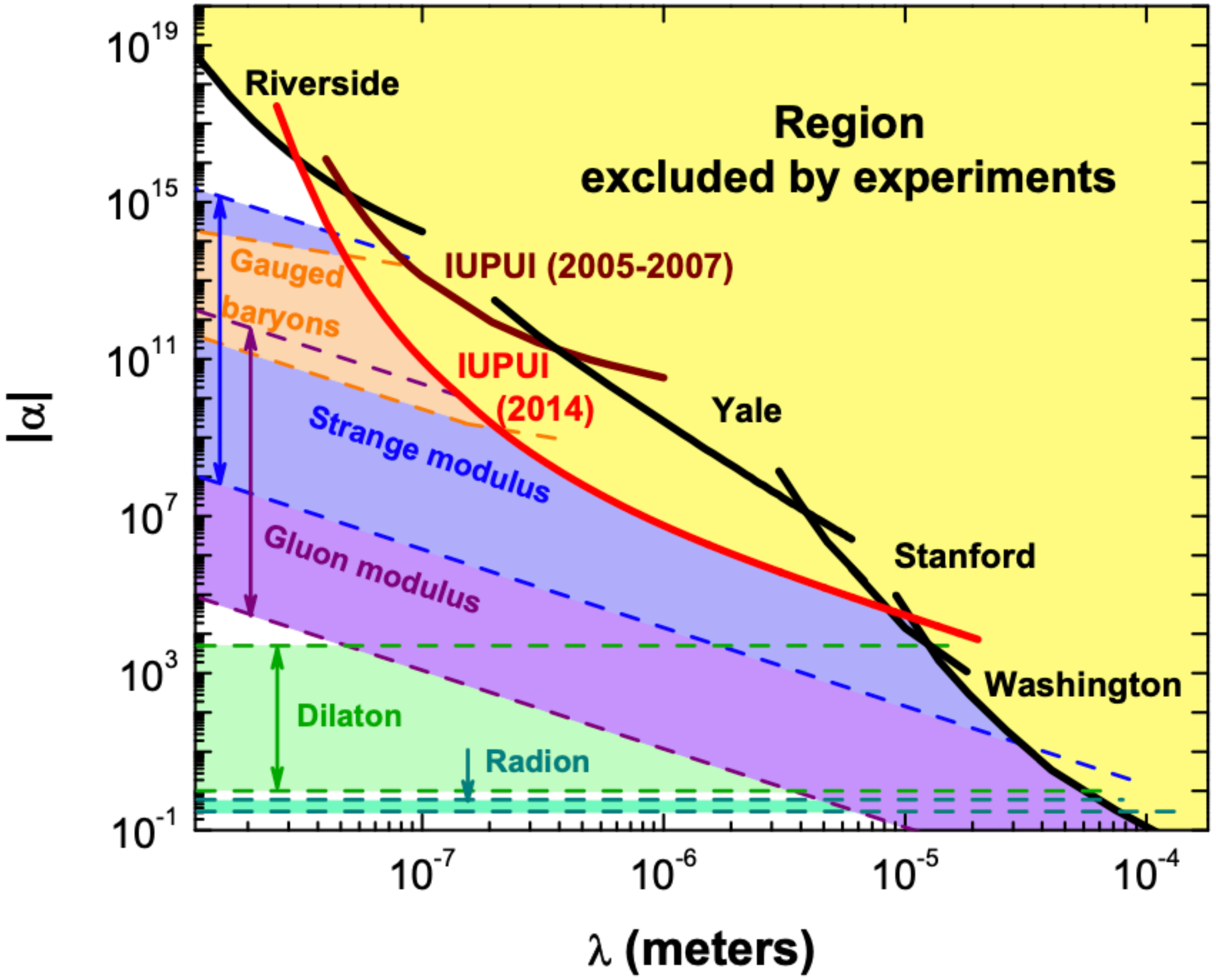

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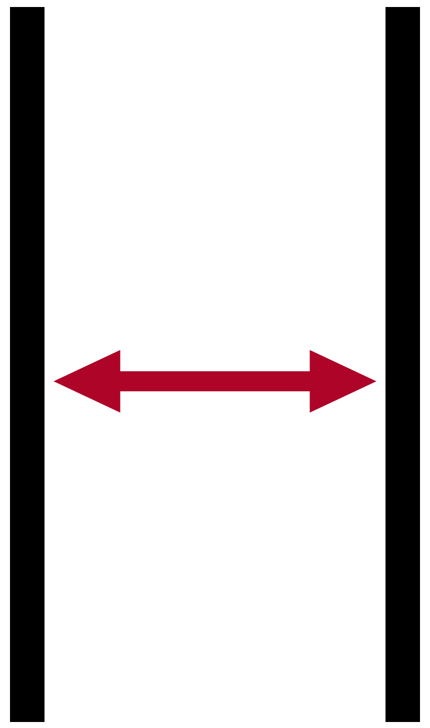

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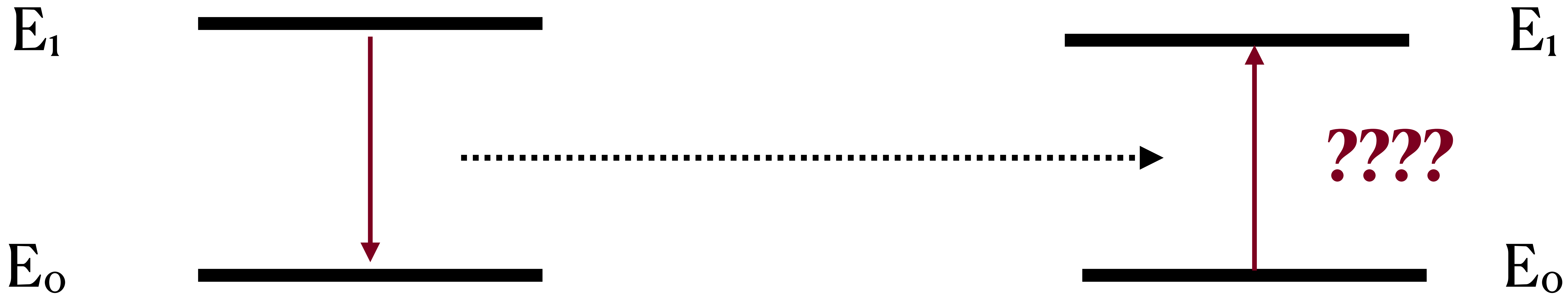
Progress?

Outline

1. Mossbauer Effect
2. Setup
3. Backgrounds
4. Sensitivity
5. Synchrotron Light Sources?
6. Conclusions

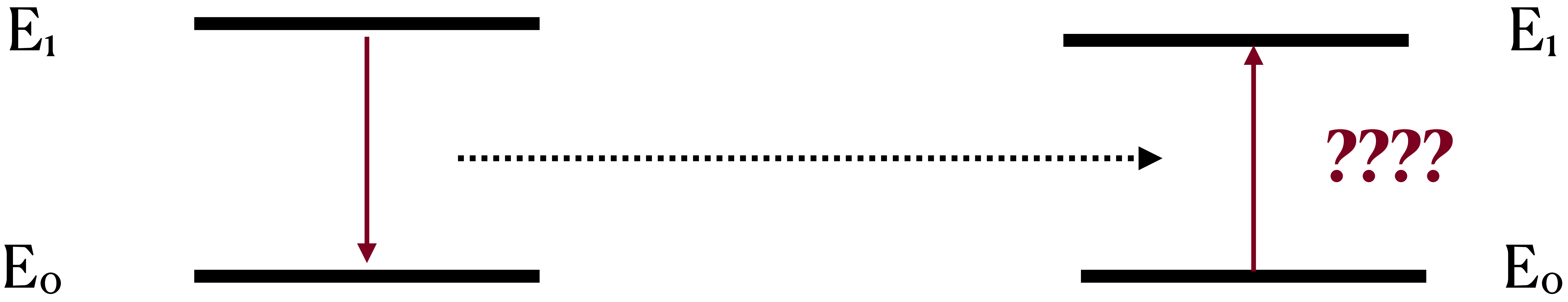
Mossbauer Effect



Excited nuclear state decays via γ emission

Can the γ be reabsorbed?

Mossbauer Effect



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Efficient reabsorption only possible on resonance

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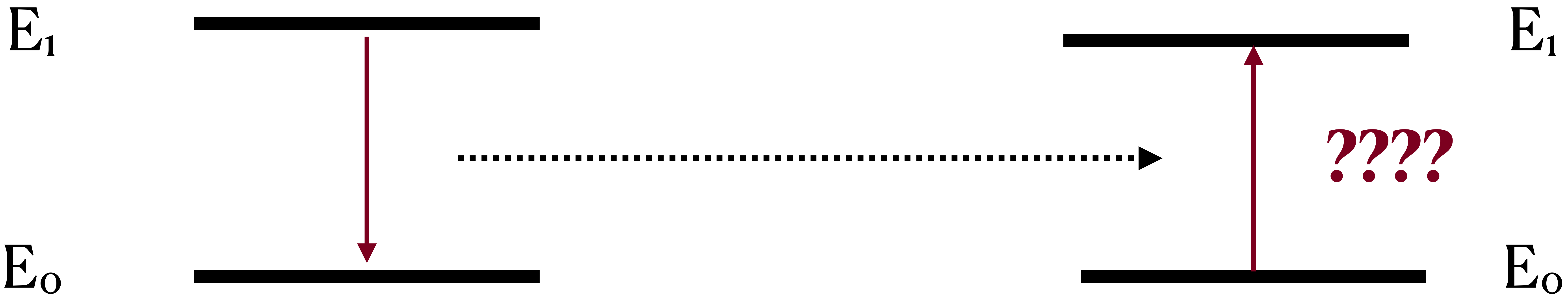

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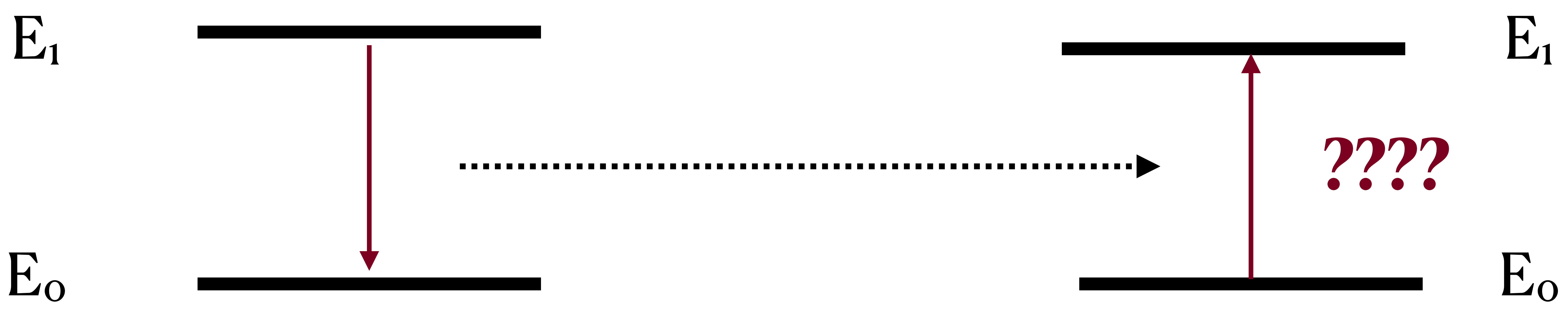

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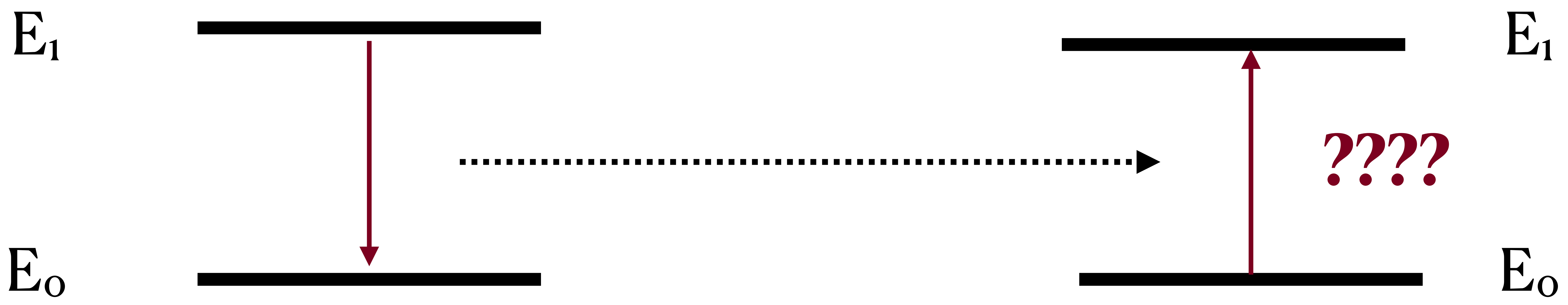
No : Recoiling nucleus takes energy, γ outside narrow width

Mossbauer Effect



Small enough E_γ, entire lattice recoils!

Negligible lattice kinetic energy - monochromatic E_γ

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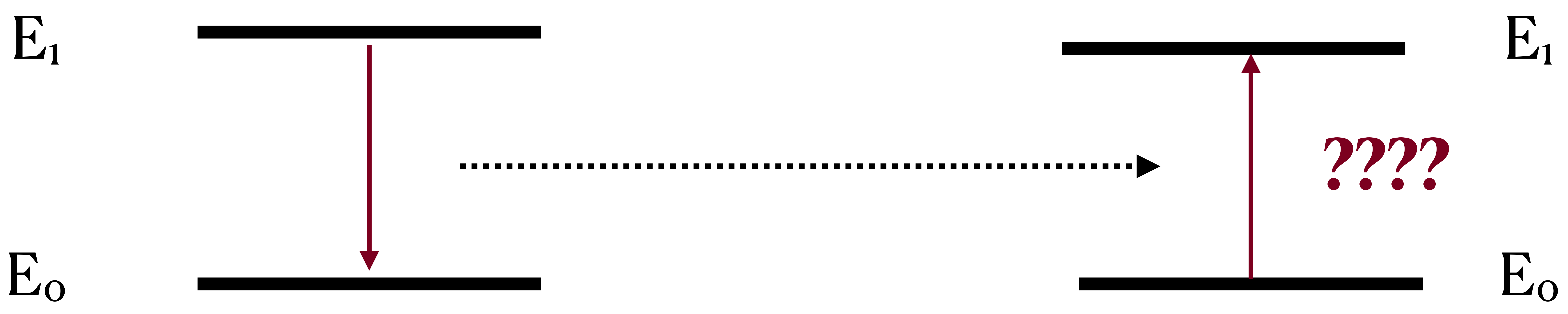

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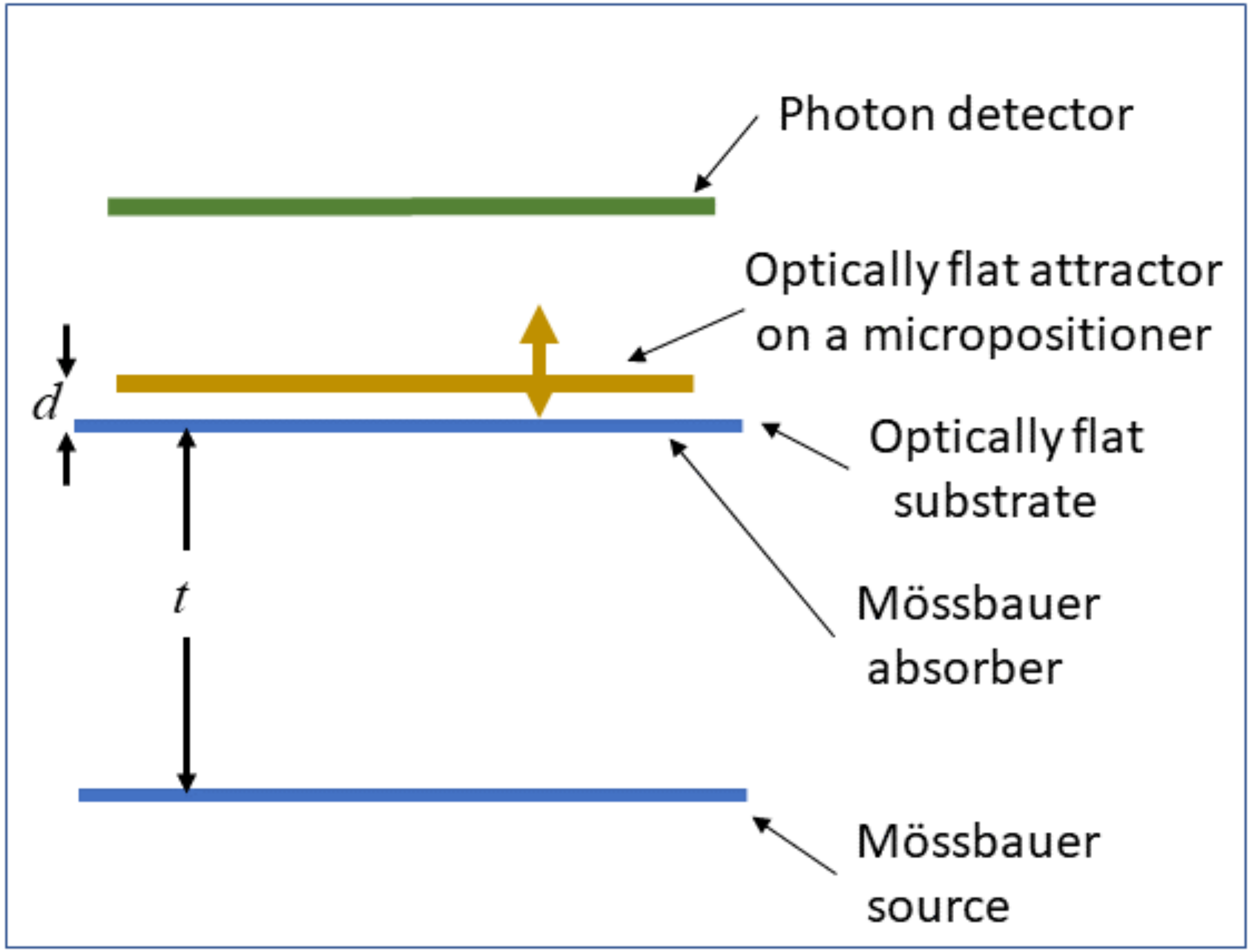
Narrow Nuclear Lines => High Sensitivity to energy shifts

Setup

$$\mathcal{L} \supset y_q \phi \bar{q} q + \frac{\phi}{f_\gamma} F_{\mu\nu}^2 + \frac{\phi}{f_g} G_{\mu\nu}^2 + \frac{\tilde{h}_{\mu\nu}}{f_T} F^{\mu}{}_{\sigma} F^{\nu\sigma} + g\phi h^2 + \frac{m_\phi^2}{2}\phi^2$$

New interaction shifts nuclear energy



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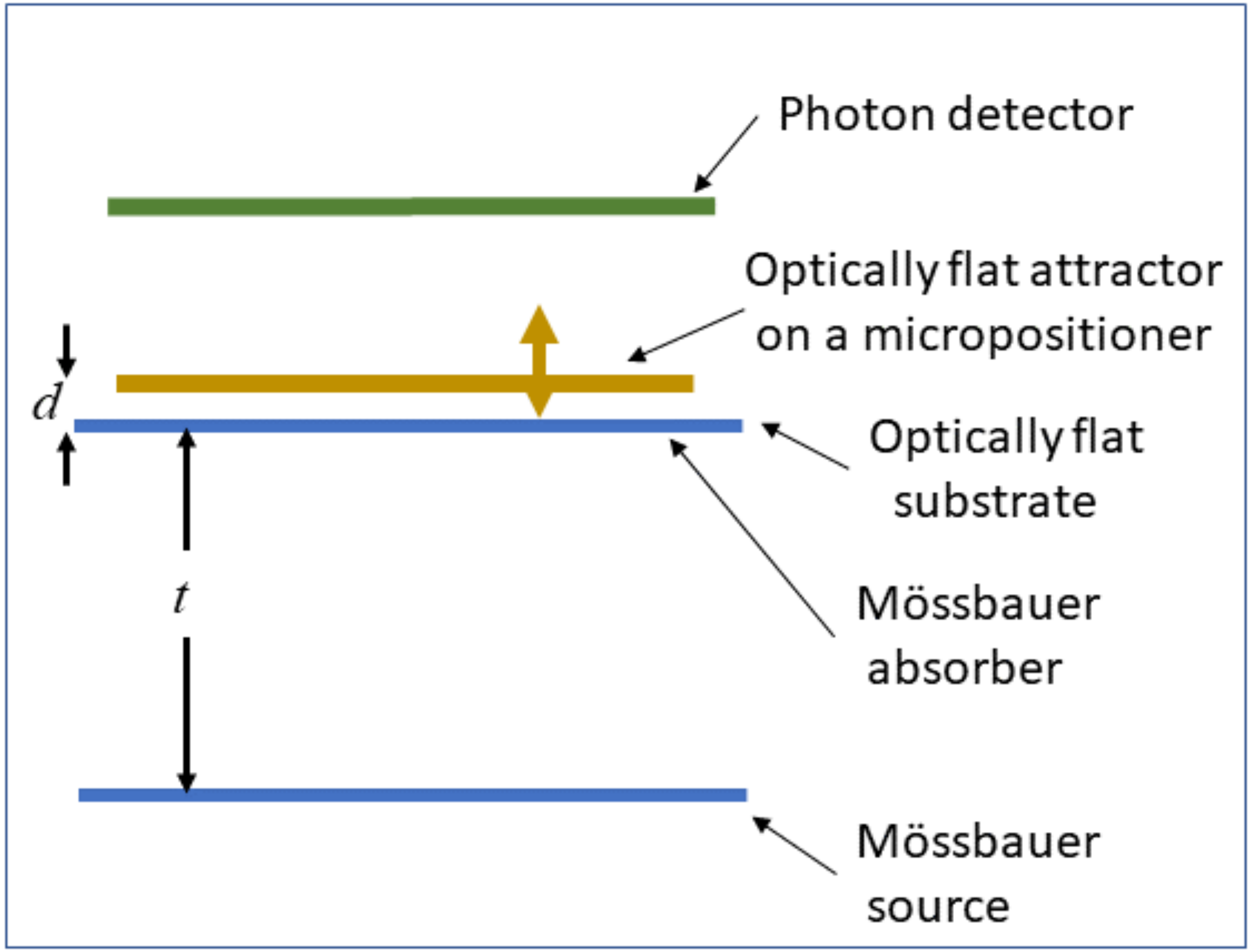
Resonant Reabsorption as a function of d



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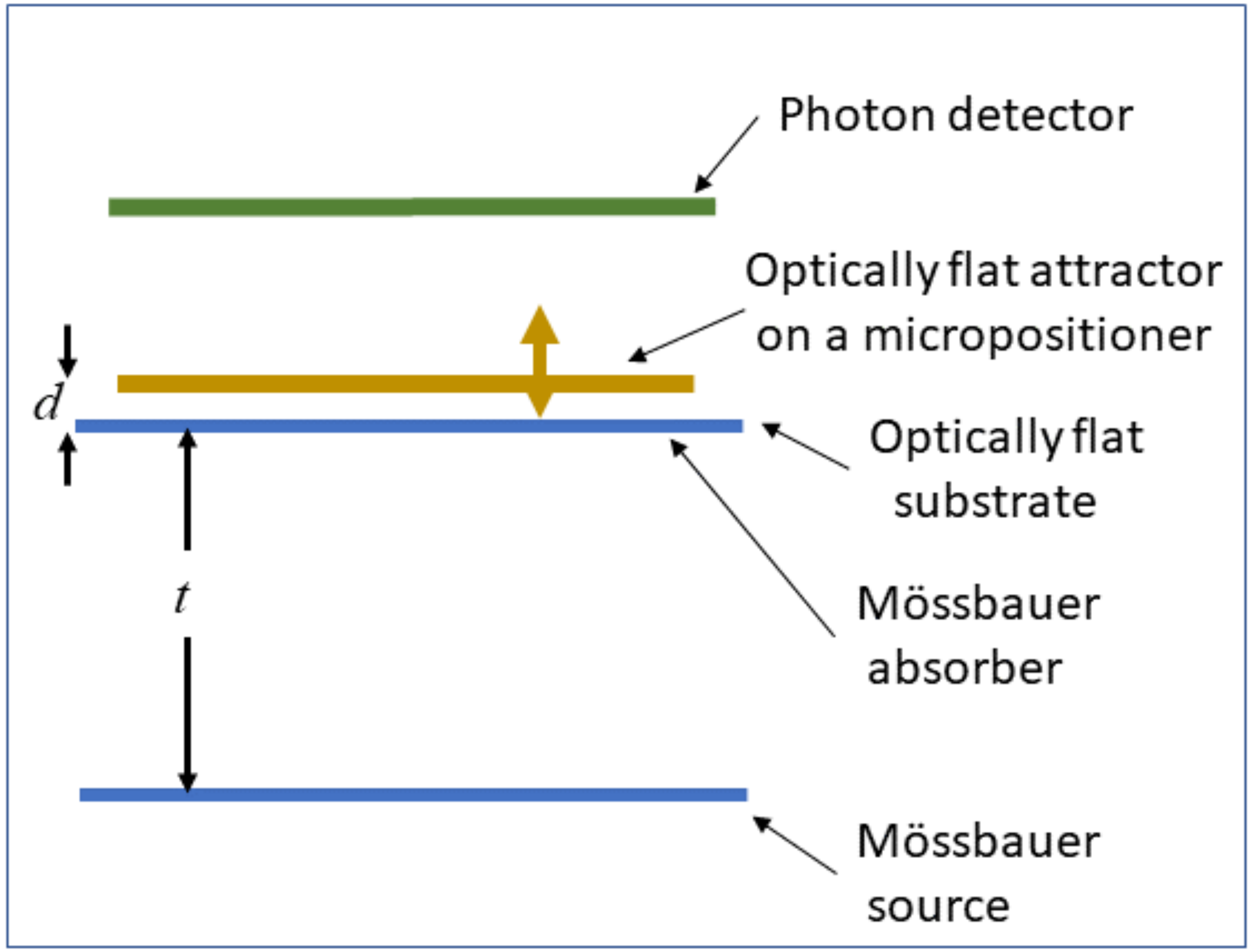
Electromagnetism?

Needs to change nuclear transition energies

Suppressed by small nuclear moments, electron shielding



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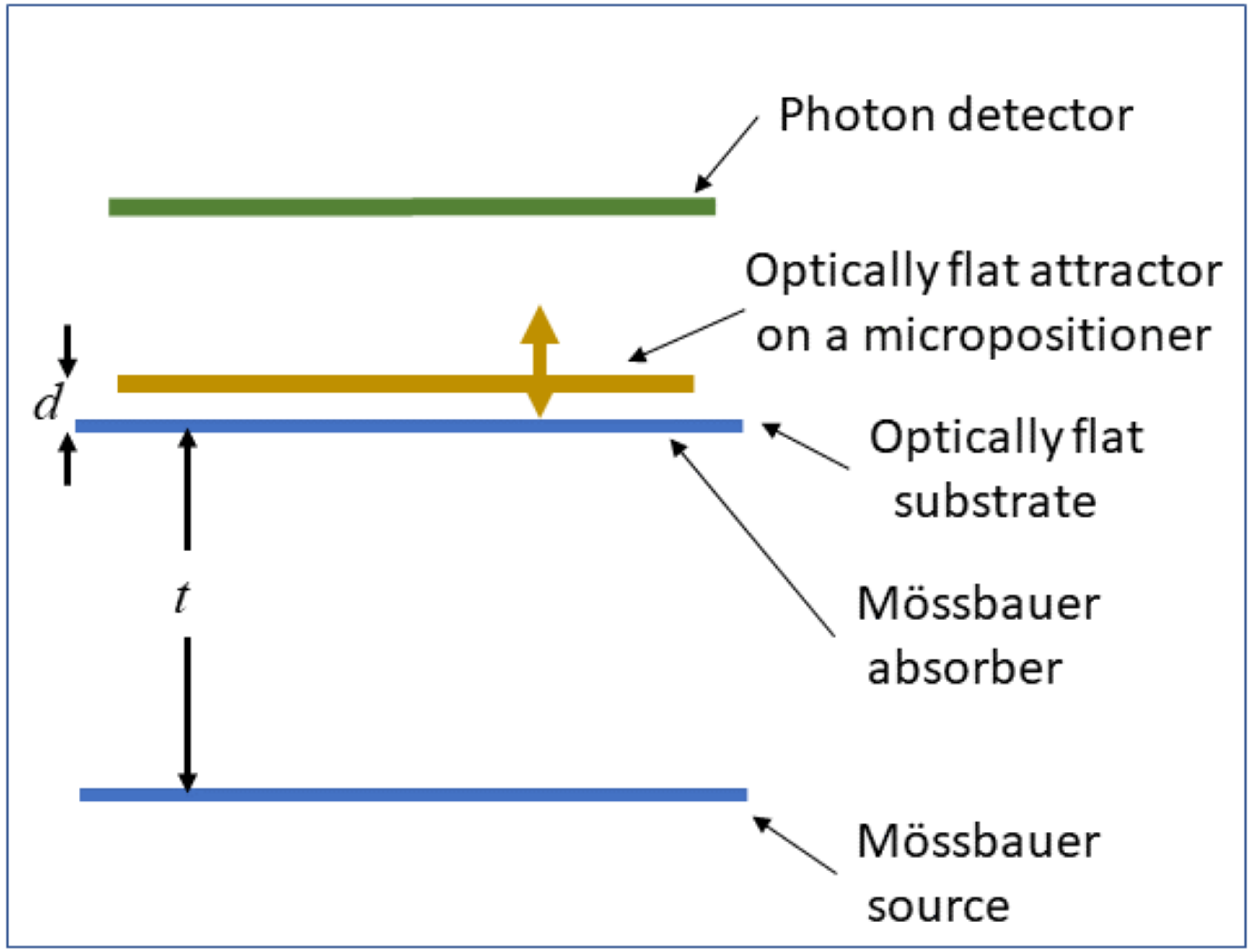

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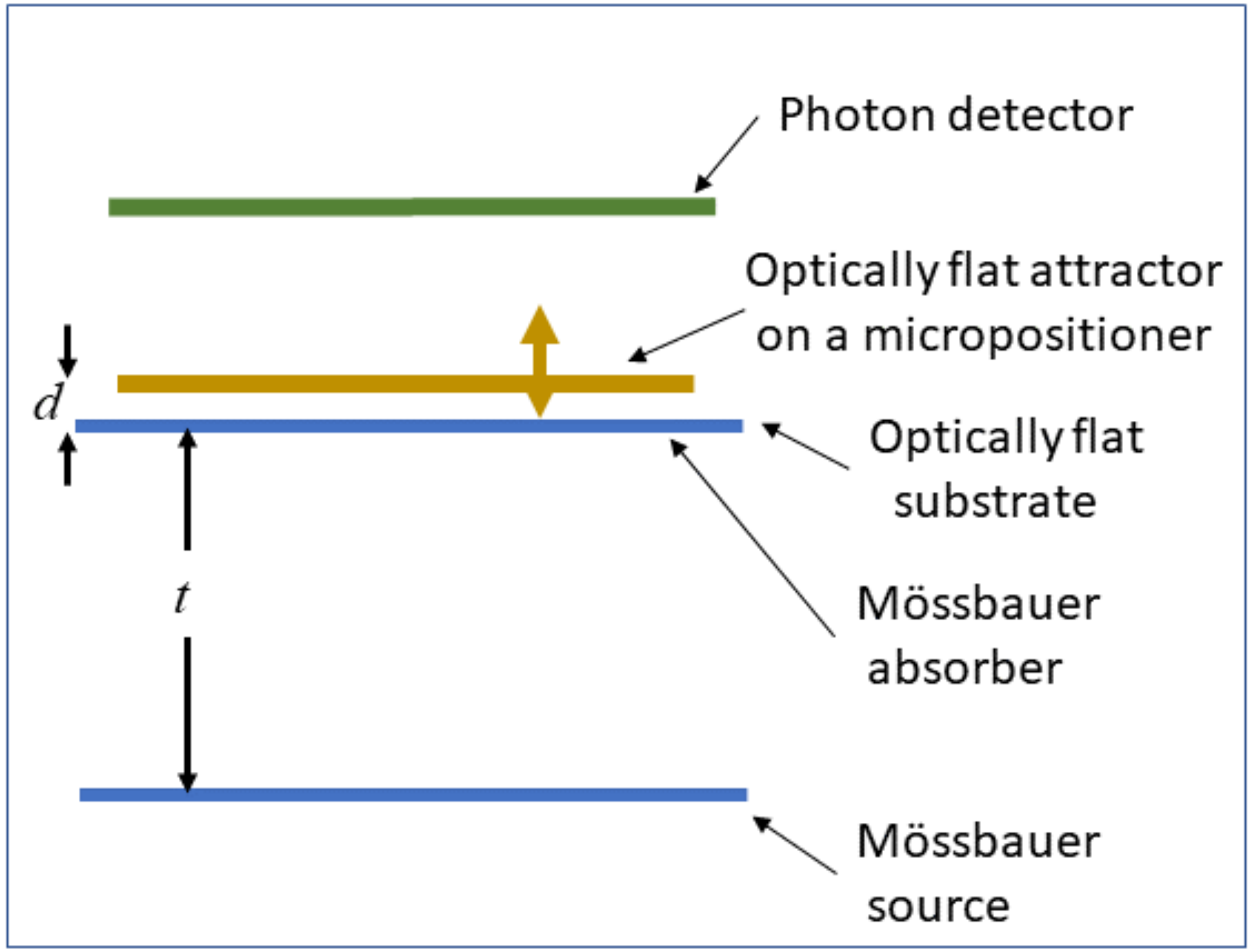

First order effects irrelevant

Leading Background: Chemical Shift from Casimir

Sensitivity

$$\mathcal{L} \supset y_q \phi \bar{q} q + \frac{\phi}{f_\gamma} F_{\mu\nu}^2 + \frac{\phi}{f_g} G_{\mu\nu}^2 + \frac{\tilde{h}_{\mu\nu}}{f_T} F^{\mu}{}_{\sigma} F^{\nu\sigma} + g\phi h^2 + \frac{m_\phi^2}{2}\phi^2$$

For given coupling, compute energy shift ΔE

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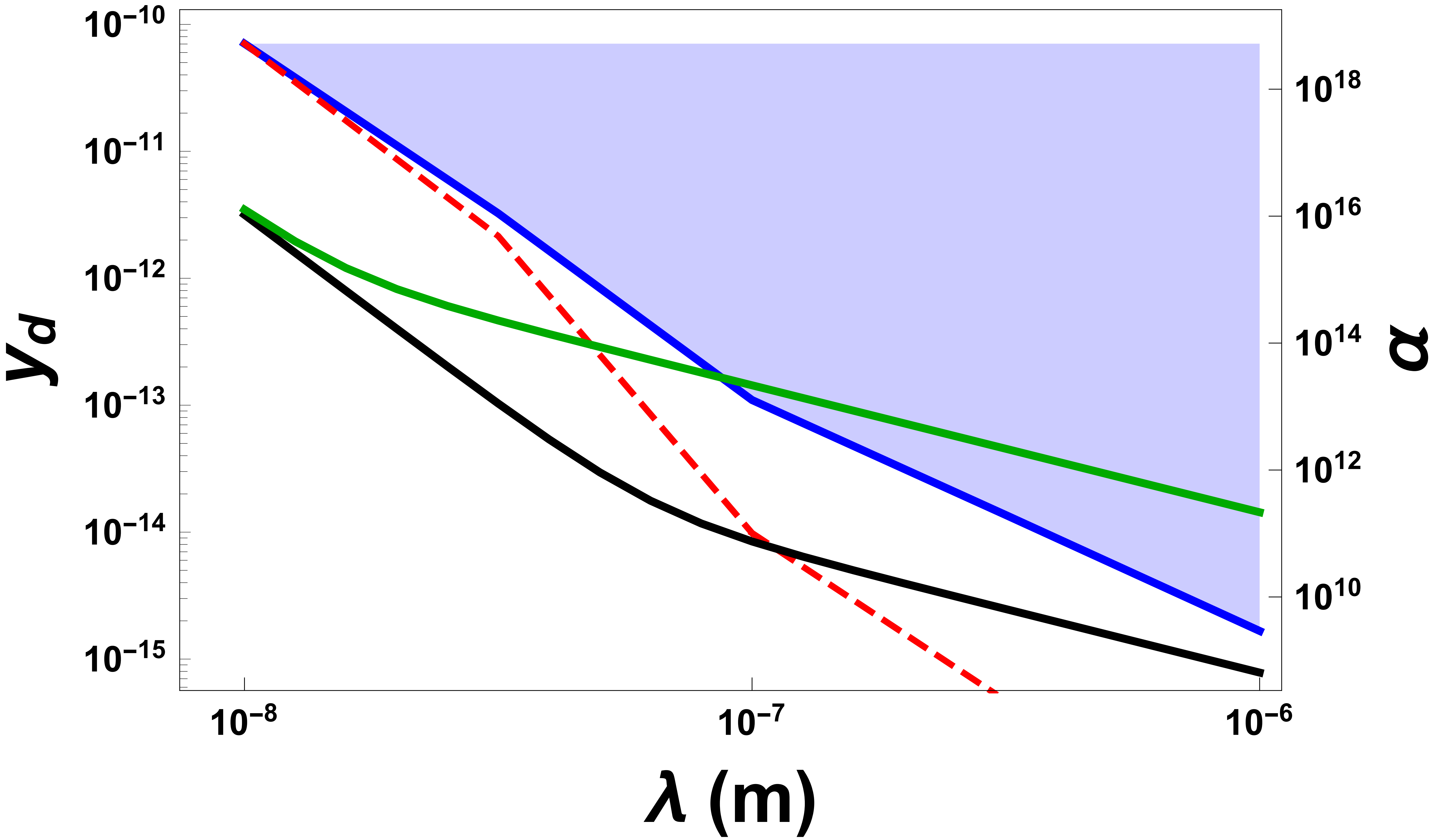
$$\Delta E = \frac{\Gamma}{\sqrt{N_\gamma}}$$

$^{57}\mathrm{Fe}$ **ΔE = 10^{-15} eV**

$^{181}\mathrm{Ta}$ **ΔE = 10^{-17} eV**

$$N_\gamma = 3 \times 10^{14}$$



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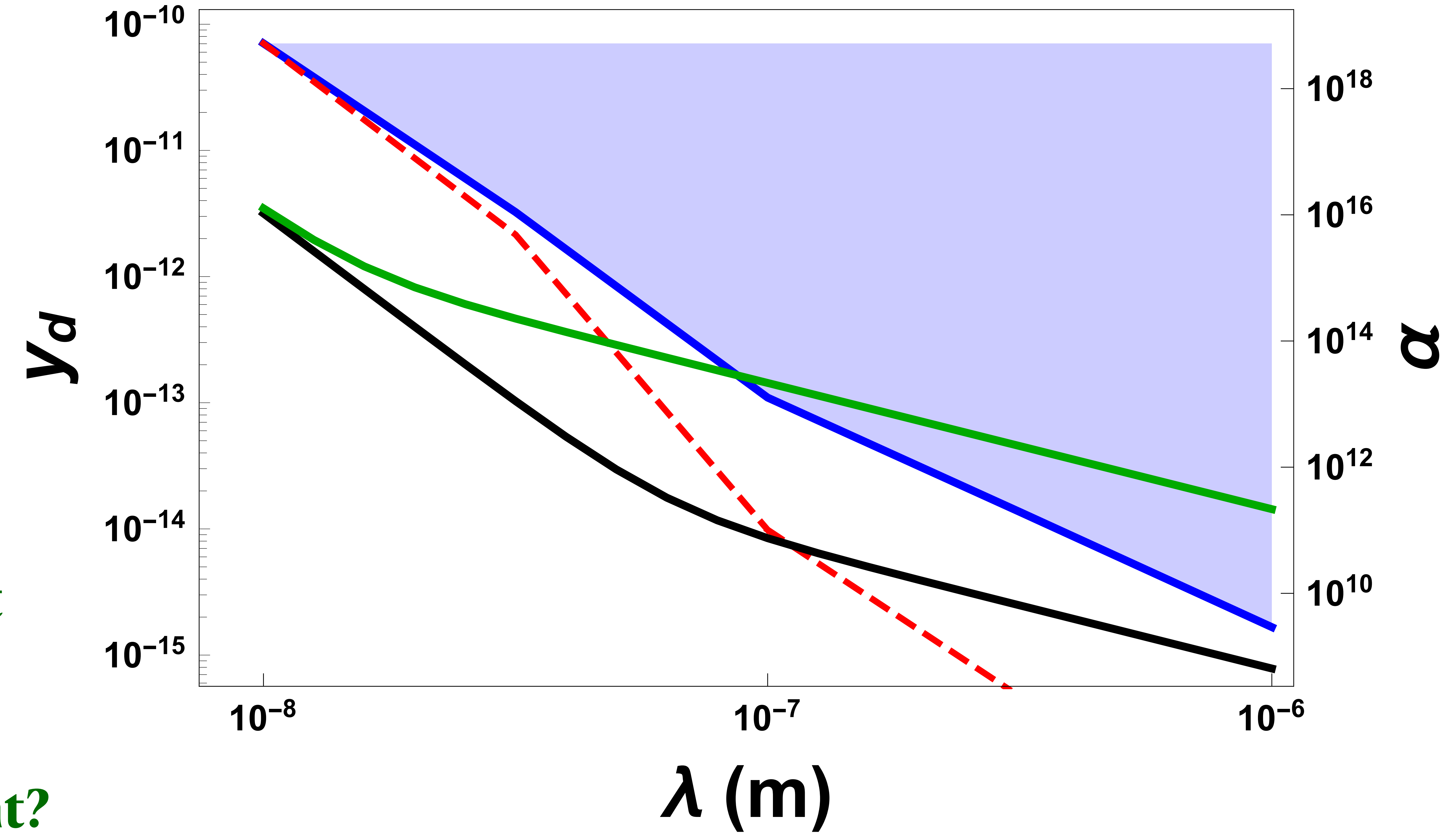
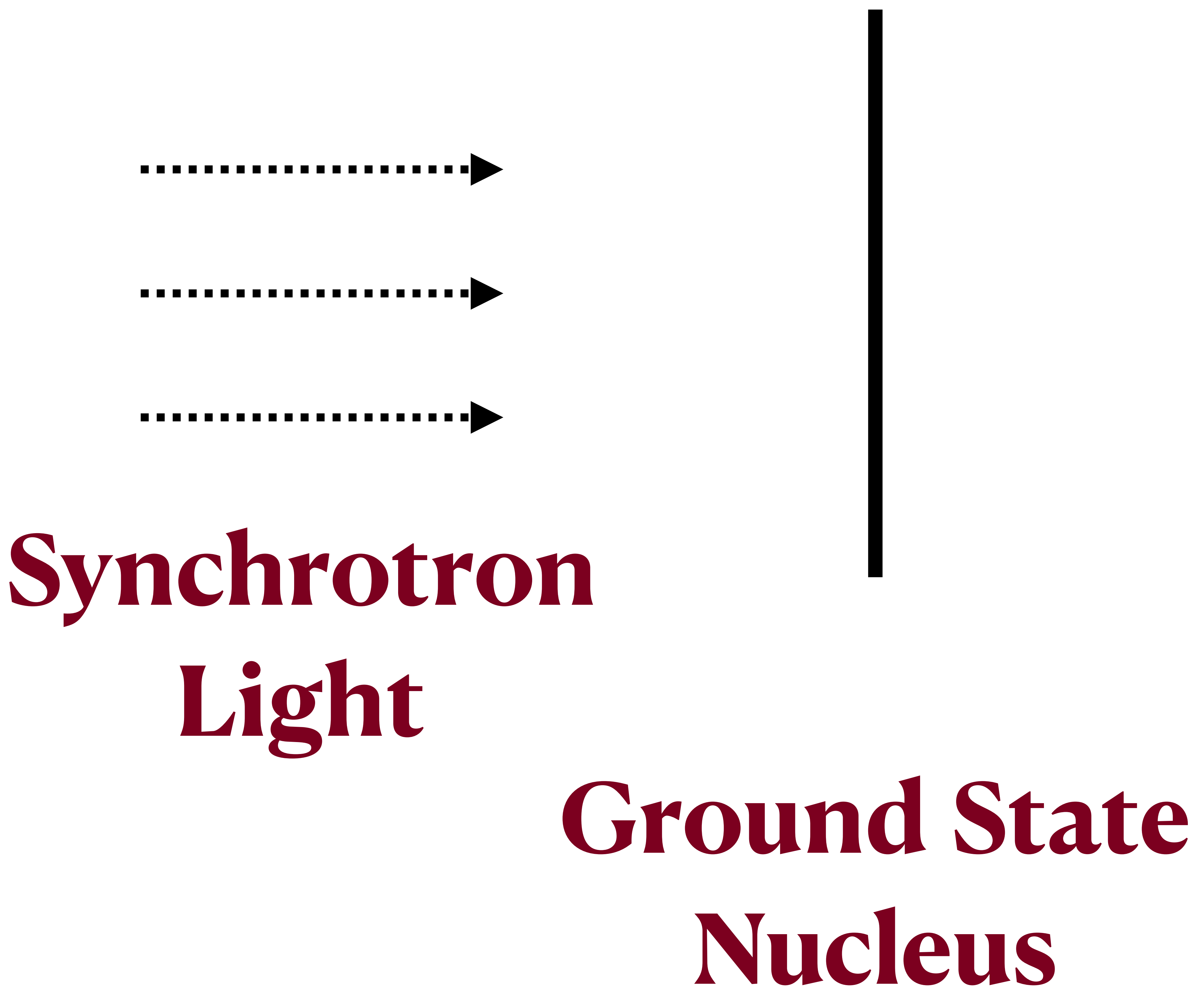
Second Order Casimir Background at shortest distances



Mitigate using differential measurement?

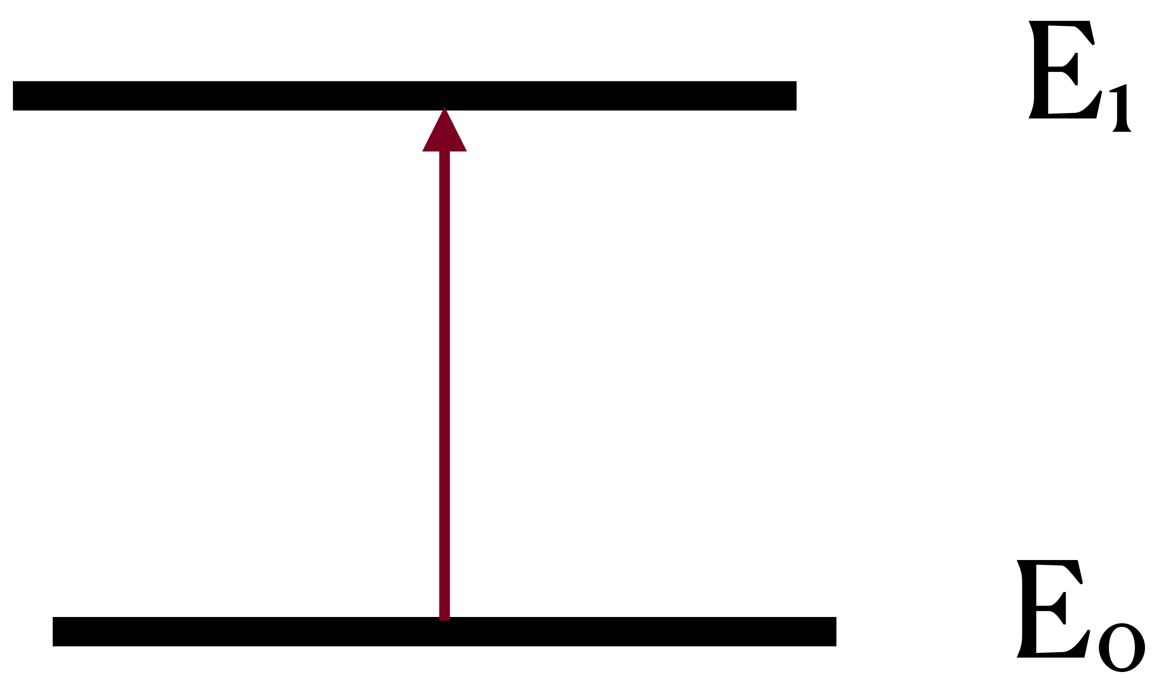
Mossbauer at Synchrotron Source



Mossbauer at Synchrotron Source

Synchrotron Light

Ground State Nucleus



All the nuclei are coherently driven to excited state

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Synchrotron
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Ground State
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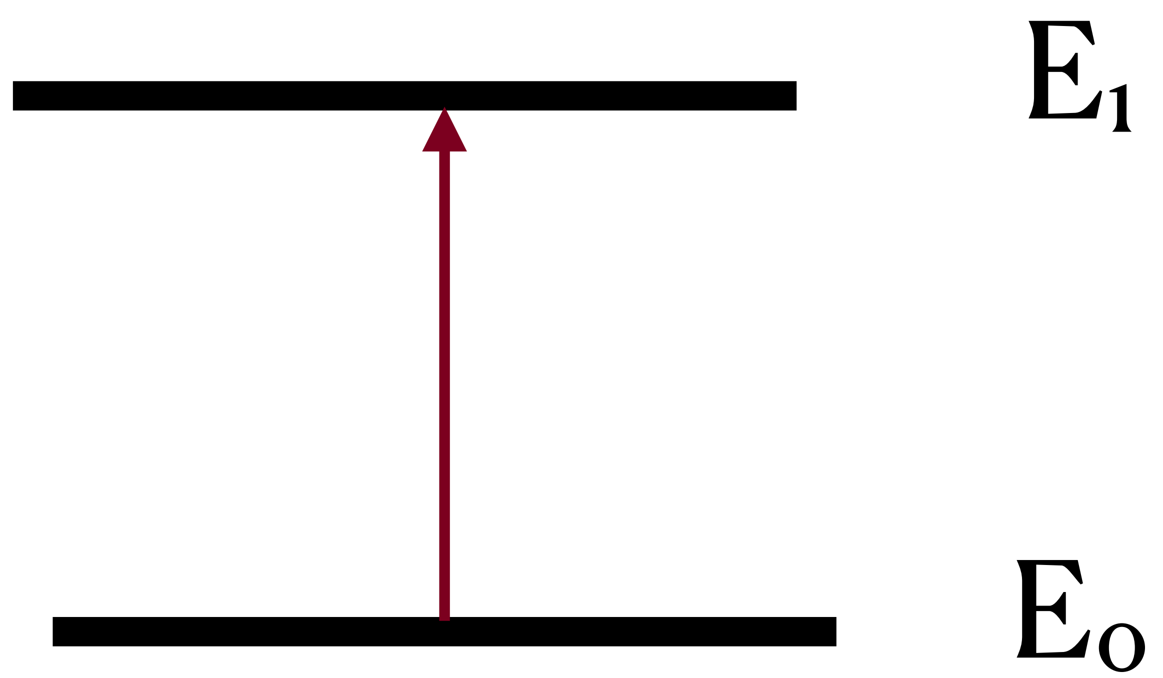

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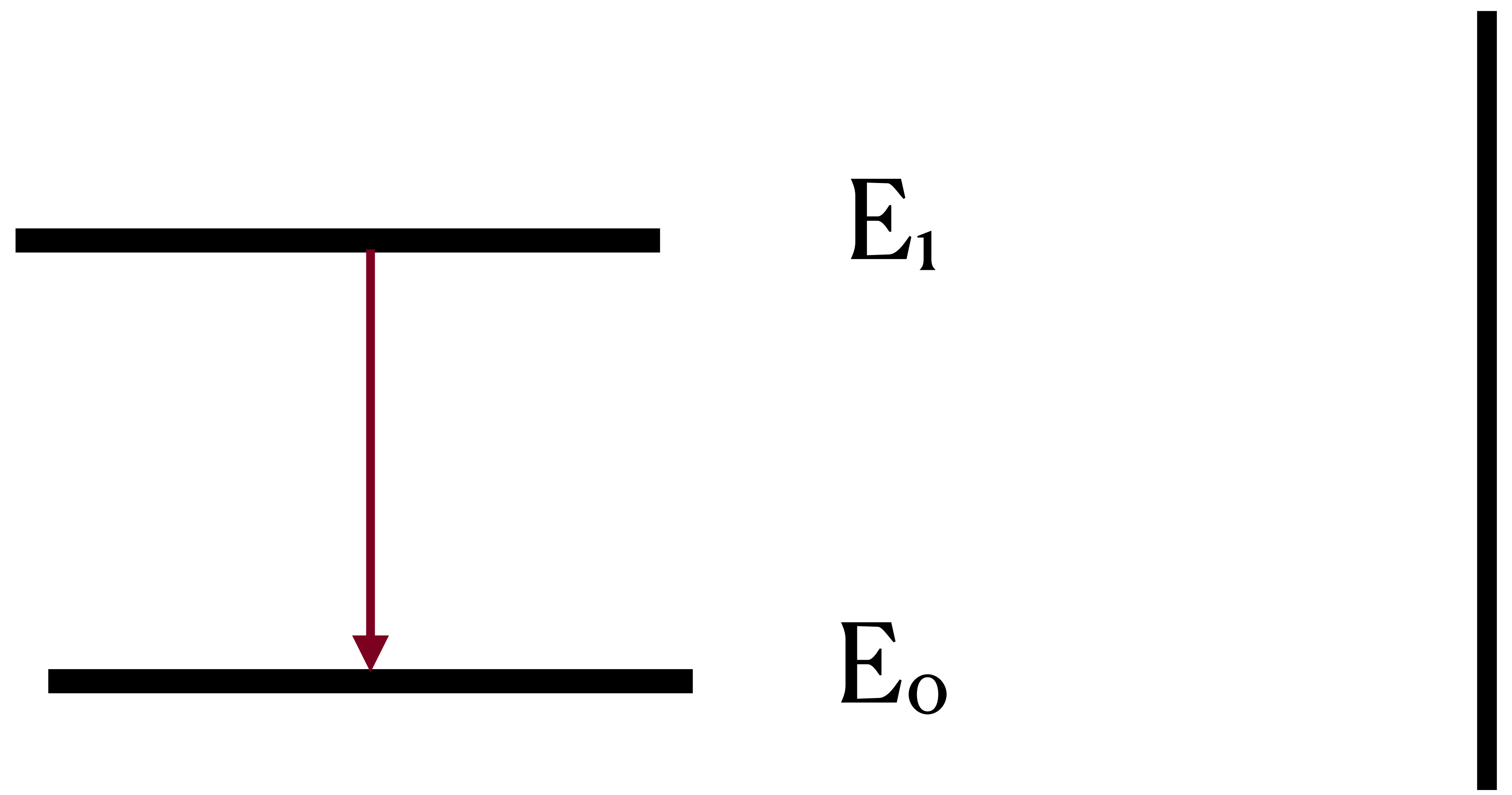
Short Pulse << Lifetime of State

Mossbauer at Synchrotron Source

Well after pulse, state starts to decay
Coherent initial excitation => decays in phase

Decay along forward direction amplified by in phase addition



In phase decay

Excited State

Mossbauer at Synchrotron Source

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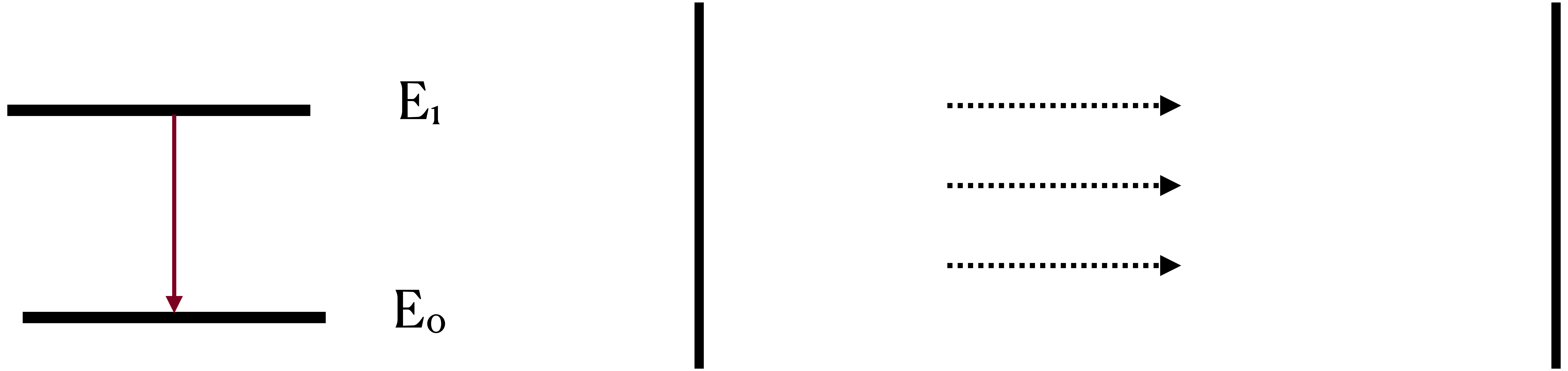
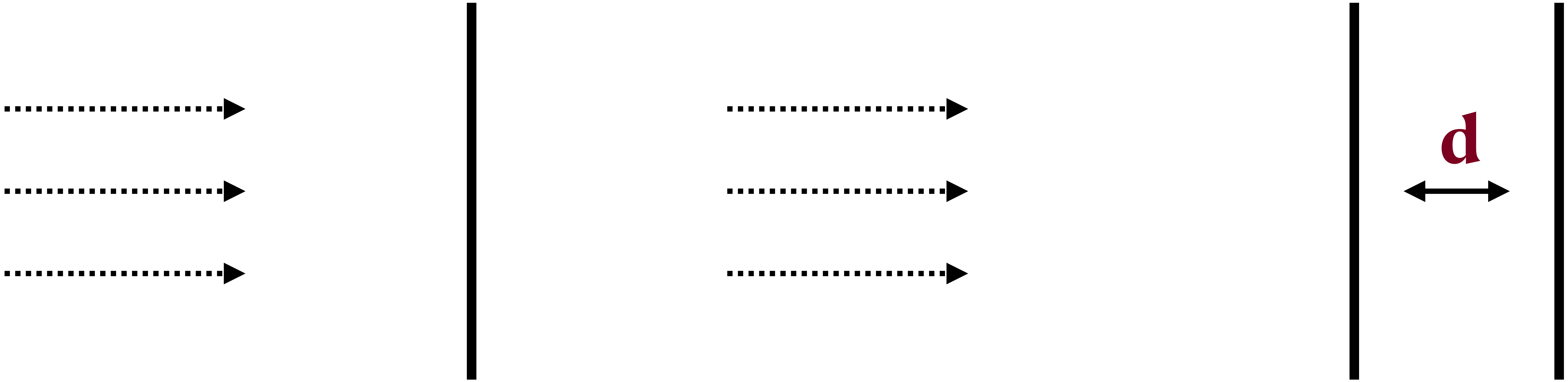


Mossbauer at Synchrotron Source

Send Synchrotron Pulse

Mossbauer at Synchrotron Source



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Well after pulse, collimated emission

Measure resonant reabsorption as a function of d

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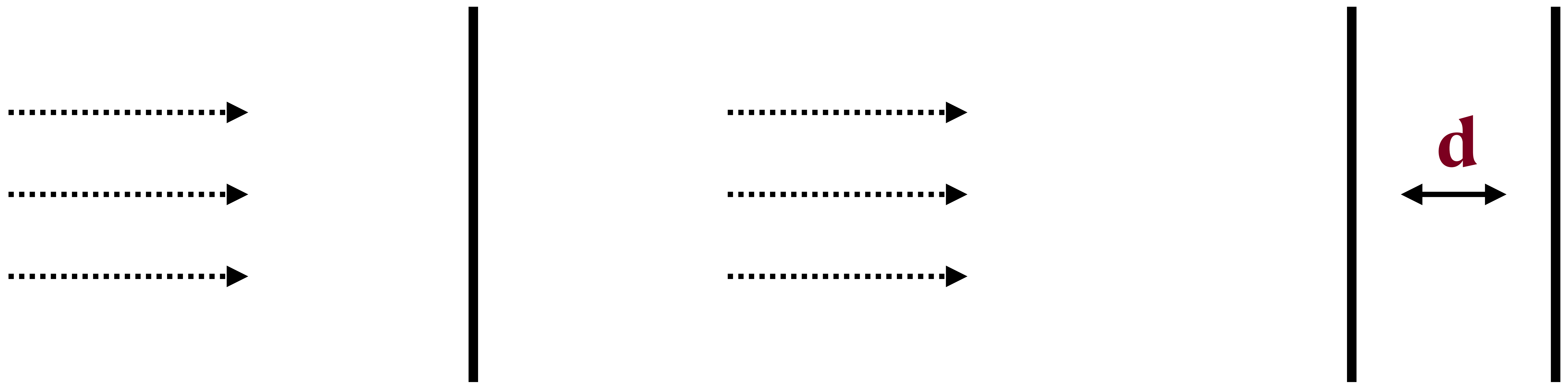

Send Synchrotron Pulse

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Measure resonant reabsorption as a function of d

Why?

Clean excitation unlike radioactive decay

May enable new class of ultra narrow Mossbauer

Conclusions

1. Mossbauer Effect seems well suited to probe short distance forces

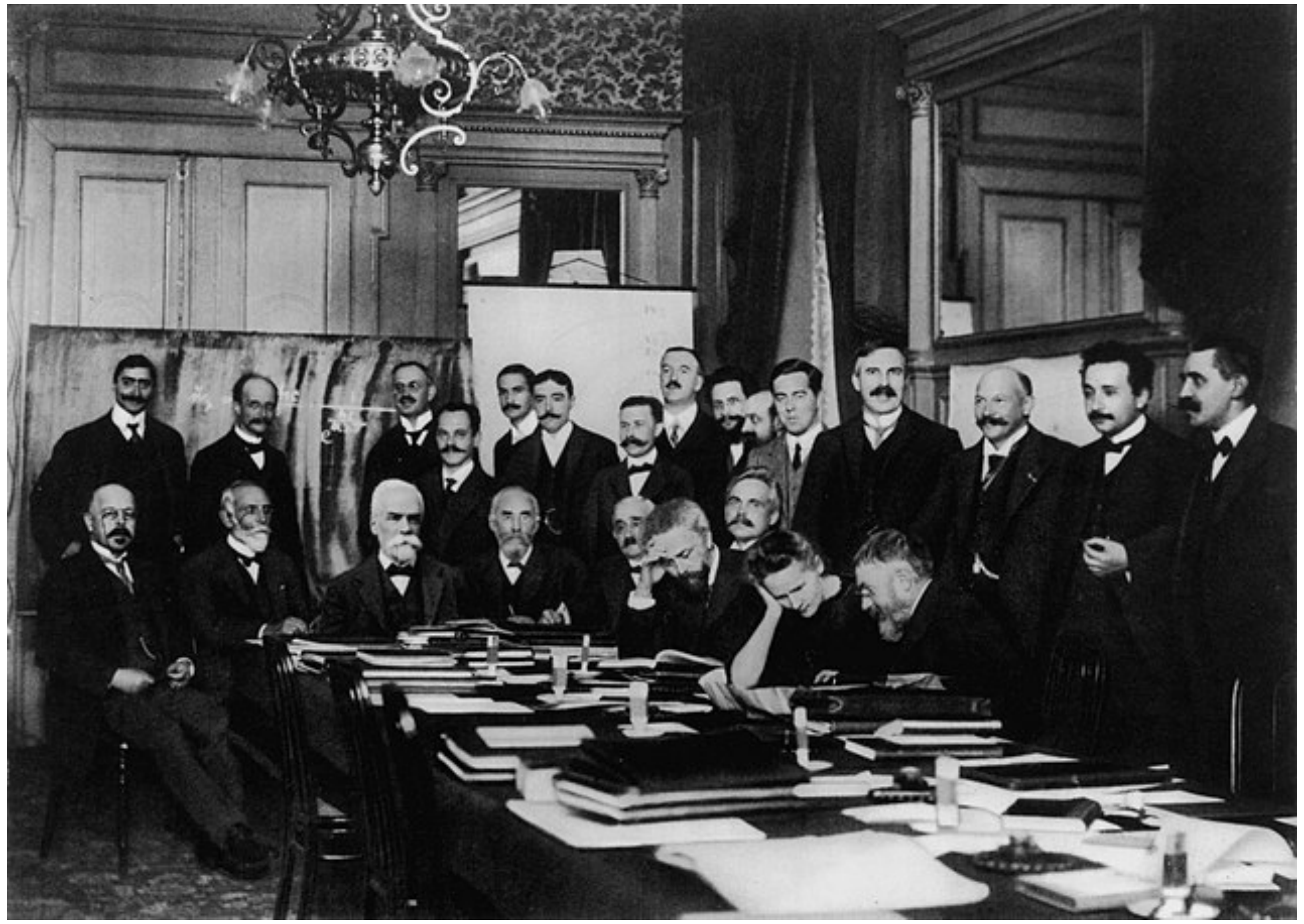
2. Natural electromagnetic background suppression

3. Ideal for scalar and tensor forces

4. Synchrotron light sources may enable new Mossbauer sources

Non Linear Quantum Mechanics

Non Linear Quantum Mechanics?



Theory built on observations in the 1900s
Why should it be "the absolute truth"?

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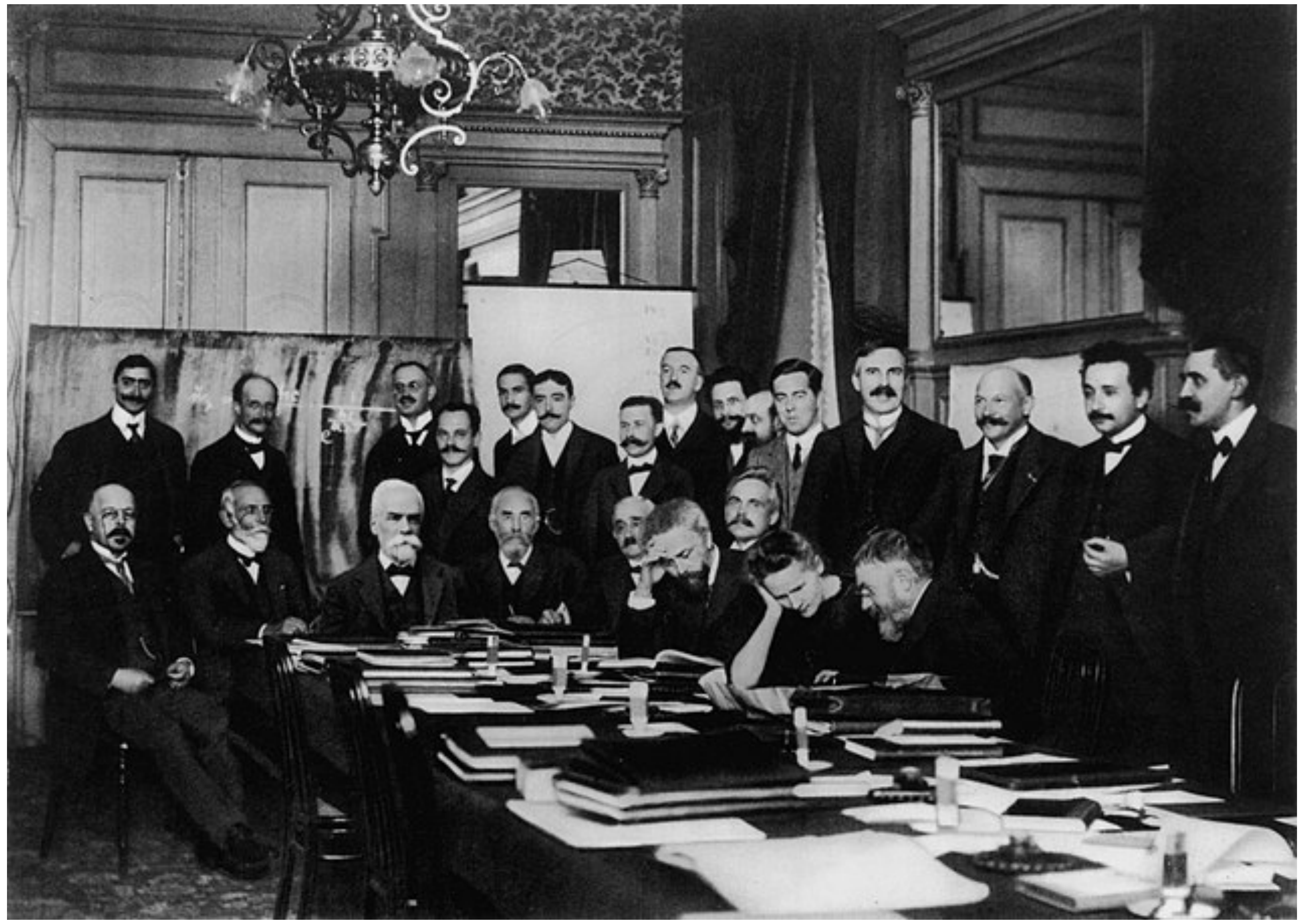

Theory built on observations in the 1900s
Why should it be “the absolute truth”?

What?

Two Postulates of Quantum Mechanics

Probability

Linearity

Which?

Probability

Probability

Finite system has a finite set of energies

Continuous observables and symmetries

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Continuous observables and symmetries

} Deterministic Observables?

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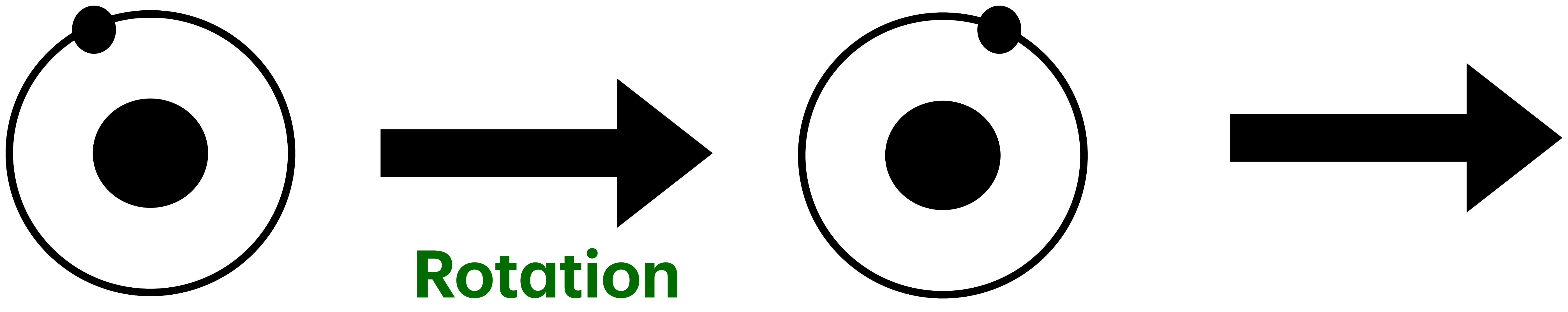

Infinite Degeneracy

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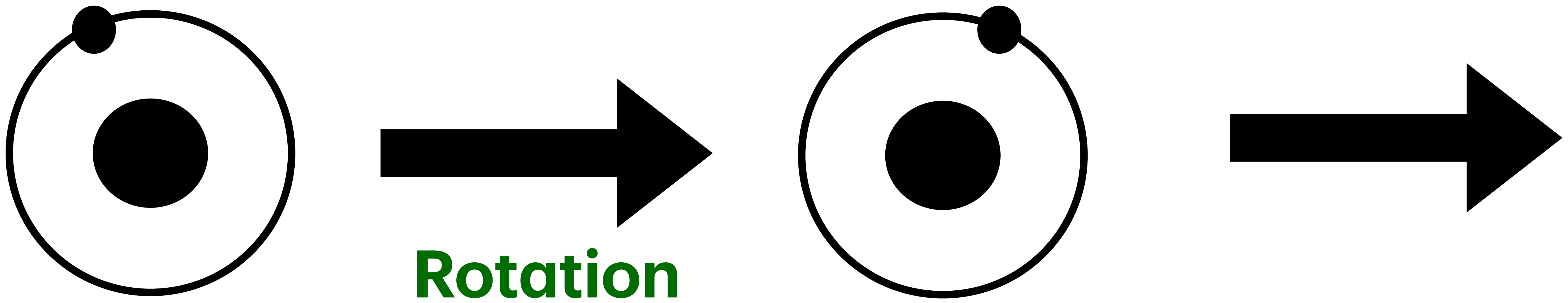

Infinite Degeneracy

Quantum Mechanics

Sacrifice Determinism.

Preserve finite set of energy states, continuous symmetries and observables

Bell Inequalities, Kochen-Specker, SSC Theorems

Causality and Entanglement

Trial Non-Linear Term

$$i\frac{\partial \Psi}{\partial t} = H_L \Psi + \epsilon \left(\Psi^2 + \Psi^{*2} \right) \Psi$$

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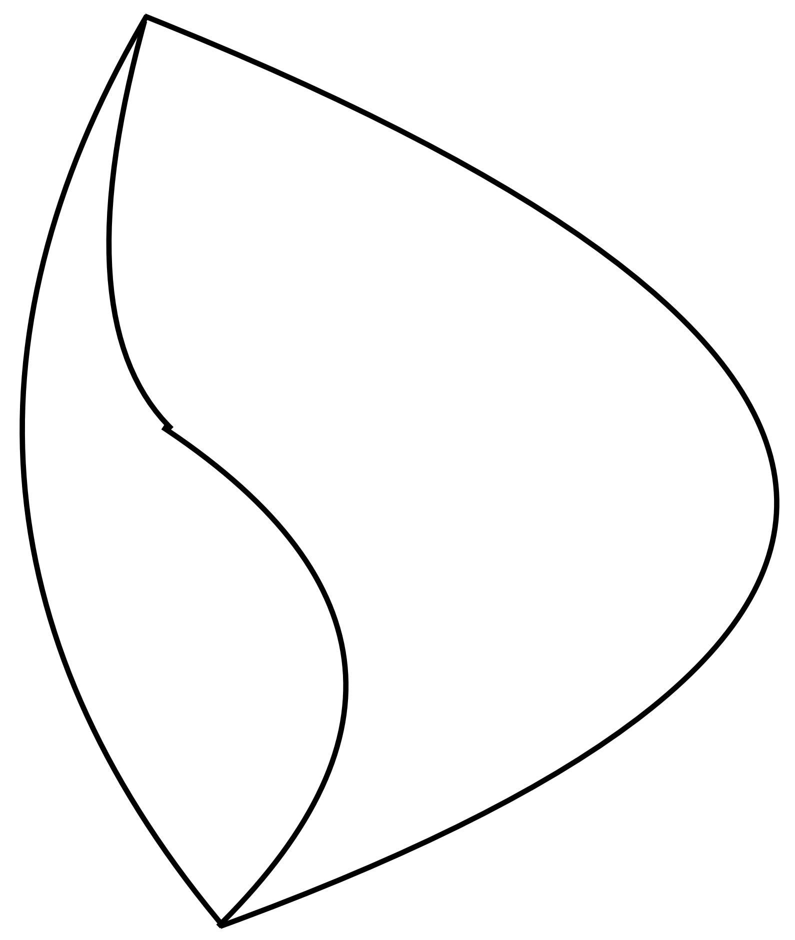
Apply some local operation on x: $a_i(x)$ -> U $a_i(x)$

Does it instantly change the time evolution of y?

YES
Not causal

Linearity

Electron Coupled to Electromagnetism



Electron paths do not interact via electromagnetism

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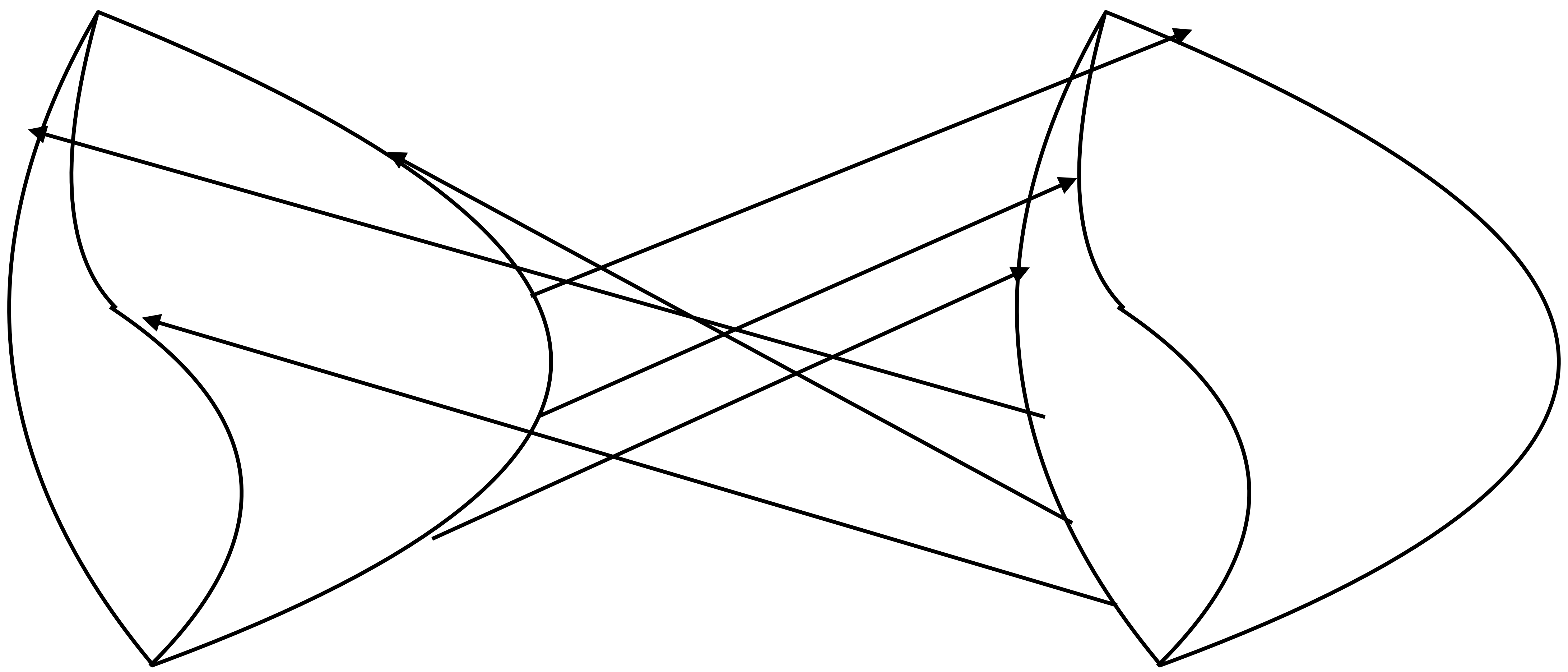

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Paths of two electrons interact causally (QFT)

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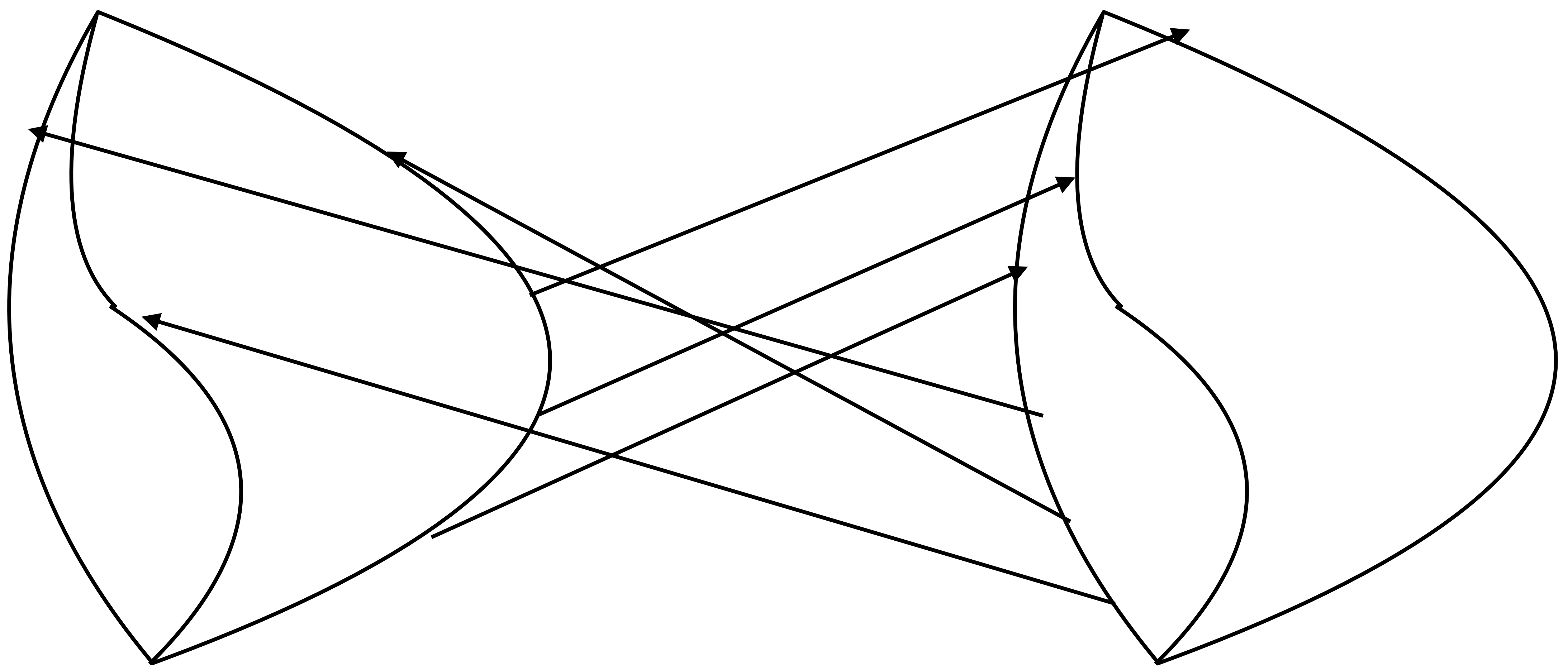

Electron paths do not interact via electromagnetism

Paths of two electrons interact causally (QFT)

Why can't path talk to itself?

Natural Language:
Quantum Field Theory

The Framework

The Schrodinger Picture of Quantum Field Theory

$$|\chi(t)\rangle$$

Quantum State of Fields (e.g. in Fock states)

$$\phi(x)$$

Time Independent Operators

$$H = \int d^3x\, \mathcal{H}\left(\phi(x), \pi(x)\right)$$

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$$S = \int dt\, \left(i\langle\chi|\dot{\chi}\rangle - \langle\chi|H|\chi\rangle\right)$$

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Yukawa $H \supset \int d^3x \, y \, \phi(x) \, \bar{\Psi}(x) \, \Psi(x)$

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Non-linearities in the operators but not in the state

The Framework

Yukawa $H \supset \int d^3x \, y \, \phi(x) \, \bar{\Psi}(x) \, \Psi(x)$

Linear QFT: $S \supset \left(\int d^3x \, y \, \langle \chi(t) | \phi(x) \, \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$

Non-Linear QFT: $S_{NL} \supset \epsilon \left(\int d^3x \, \langle \chi(t) | \phi(x) | \chi(t) \rangle \, \langle \chi(t) | \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$

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Obeys all the rules

Higher order in states - leads to state dependent quantum evolution

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Analyze non-linearity perturbatively

Perturbation Theory

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = \left(y\phi + \epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

$$i\frac{\partial|\chi\rangle}{\partial t} = H|\chi\rangle$$

At zeroth order, this is just standard QFT

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Perform standard QFT on this background field to compute first order correction

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Causality: Non-linearity enters via expectation value. At lowest order, causal from QFT.
Causal background field for all higher orders

Single Particle

$$\mathcal{L} \supset y\Phi\bar{\Psi}\Psi = y\left(\phi + \tilde{\epsilon}\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

Suppose we have a ψ particle - how does its wave-function evolve?

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Straightforward Computation of Expectation Value

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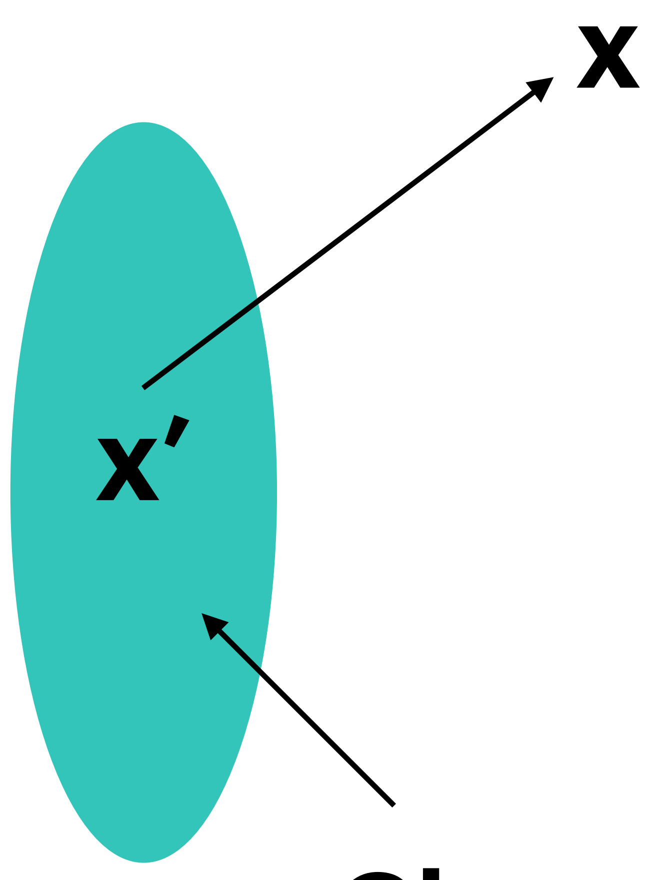

$$\langle\chi|\phi\left(x\right)|\chi\rangle = \int d^4x'\psi^*\left(x'\right)\psi\left(x'\right)G_R\left(x-x'\right)$$

Charge Density of ψ

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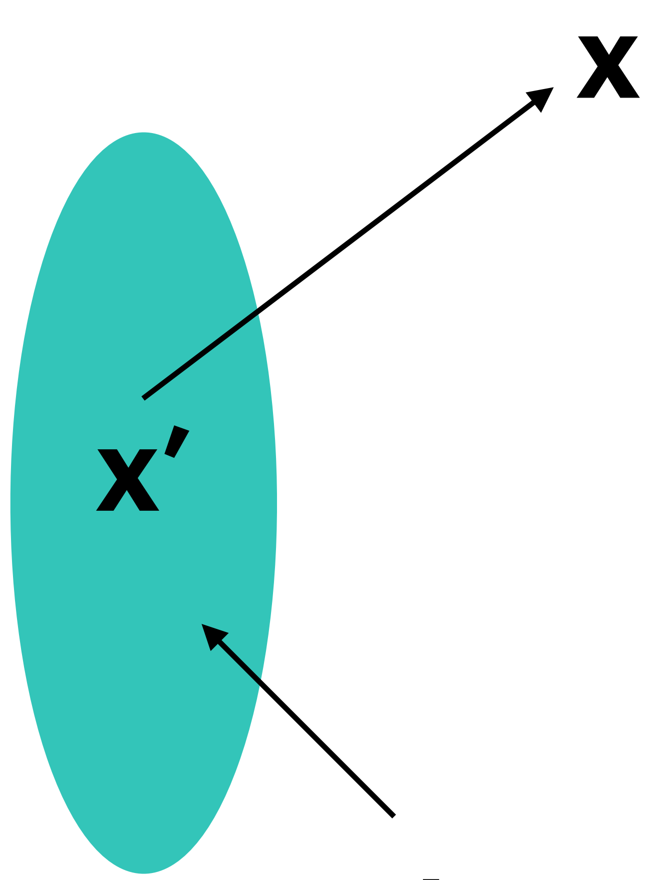

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Charge Density of ψ

Causal Green's Function

Schrodinger Equation

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Single particle equation derived from field theory
Equation depends upon theory (Yukawa, Φ^4 etc)

$$i\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = \left(H + \tilde{\epsilon}y\int d^4x'\Psi^*(x)\Psi(x')G_R(x;x')\right)\Psi(t,\mathbf{x})$$

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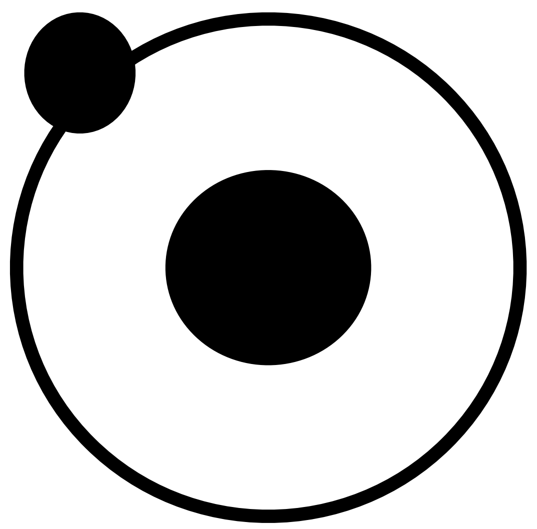

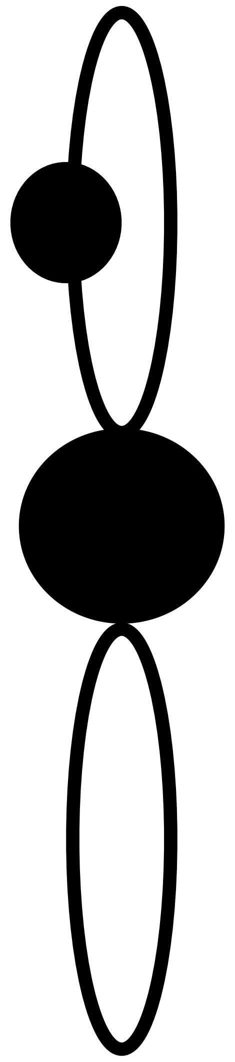

Fixed Central particle

Self interaction of wave-function breaks degeneracy of levels

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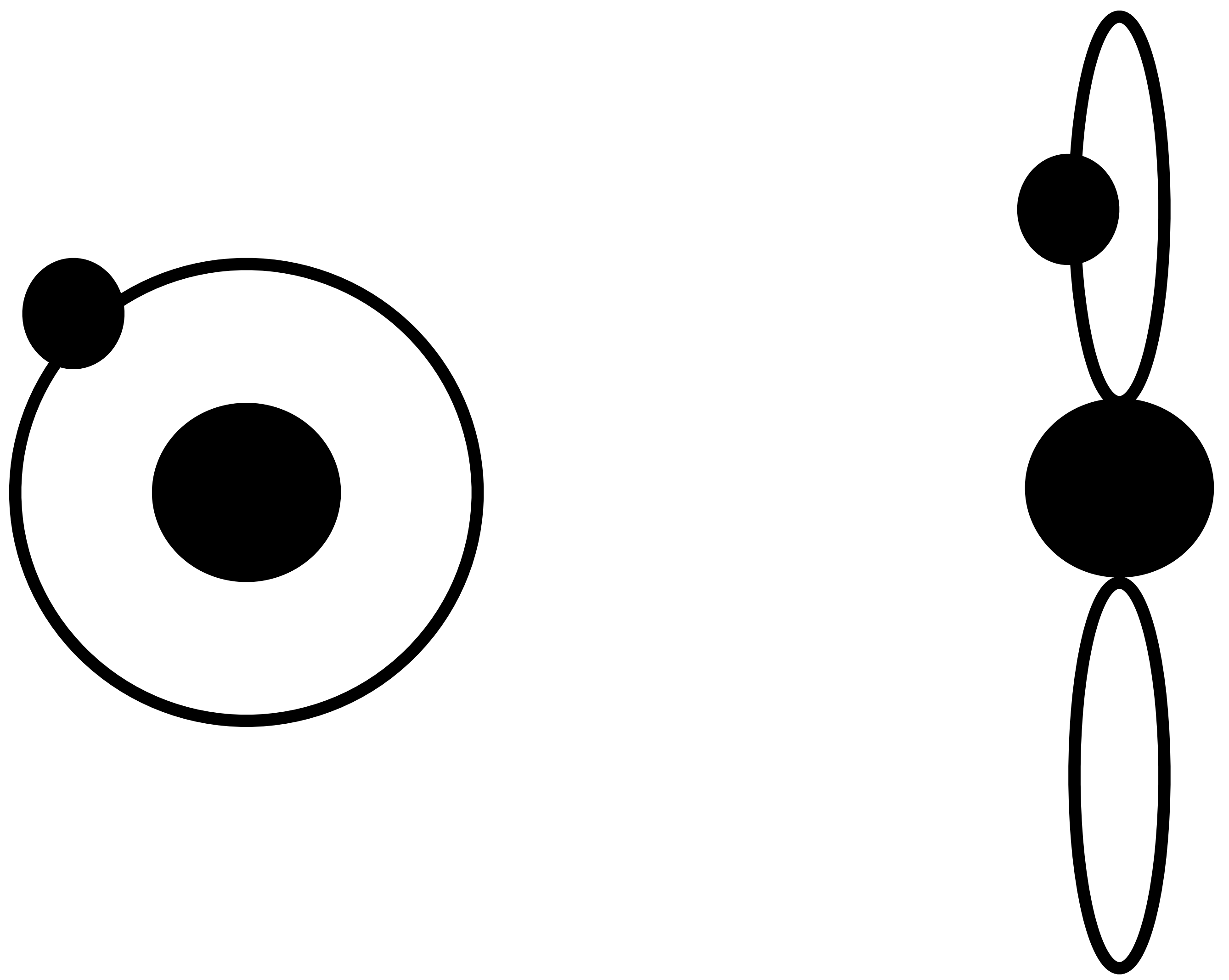
Single particle equation derived from field theory
Equation depends upon theory (Yukawa, Φ^4 etc)

$$i\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = \left(H + \tilde{\epsilon}y\int d^4x'\Psi^*\left(x\right)\Psi\left(x'\right)G_R\left(x;x'\right)\right)\Psi\left(t,\mathbf{x}\right)$$



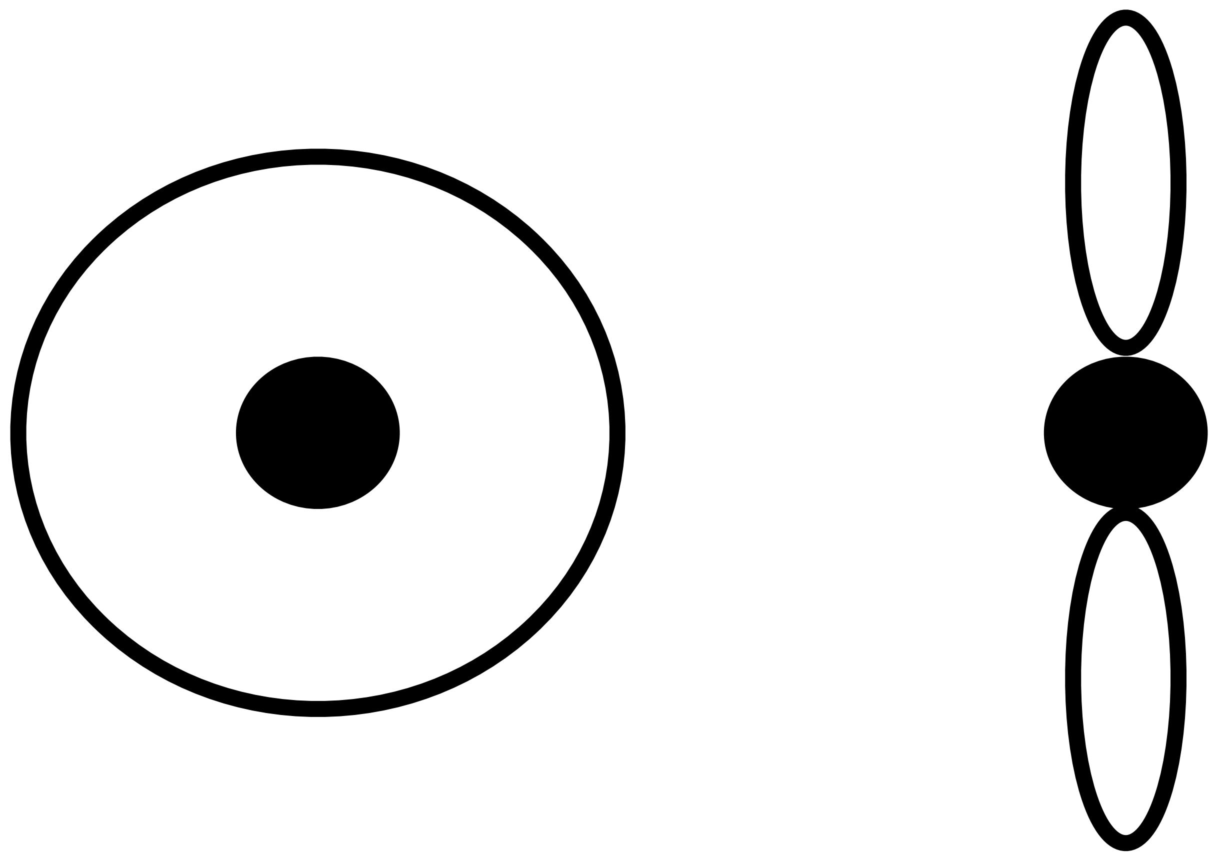
Fixed Central particle

Self interaction of wave-function breaks degeneracy of levels

Hermitean Form of Hamiltonian implies conserved norm

Constraints

What does this do to the Lamb Shift?



$$\langle \chi | A_\mu | \chi \rangle J^\mu$$

Proton at Fixed Location

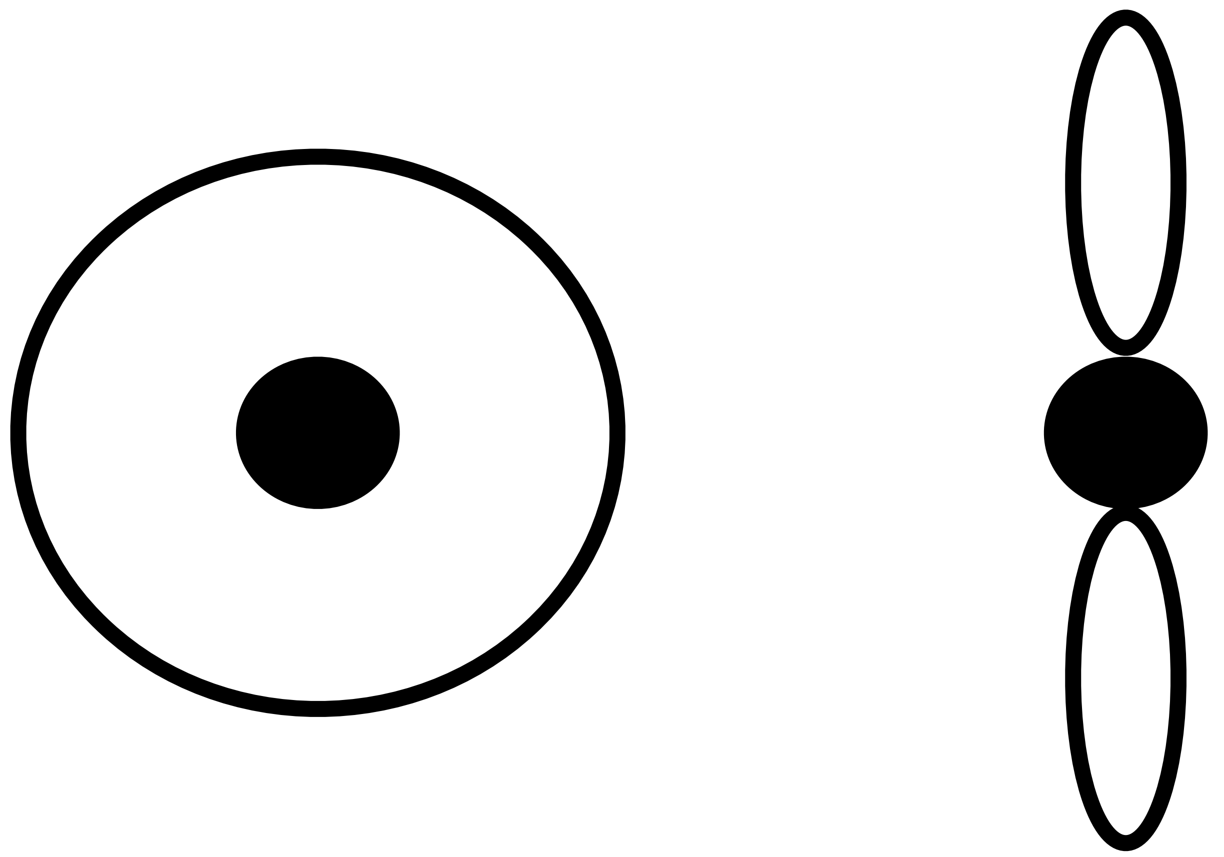
2S and 2P electron have different charge distribution

Different expectation value of electromagnetic field

Level Splitting!

Constraints

What does this do to the Lamb Shift?



$$\langle \chi | A_\mu | \chi \rangle J^\mu$$

Proton at Fixed Location

2S and 2P electron have different charge distribution

Different expectation value of electromagnetic field

Level Splitting!

BUT: Cannot decouple center of mass and relative co-ordinates

Proton wave-function spread over some region (e.g. trap size ~ 100 nm)
Expectation value of electromagnetic field diluted
In neutral atom - heavily suppressed, except at edges!

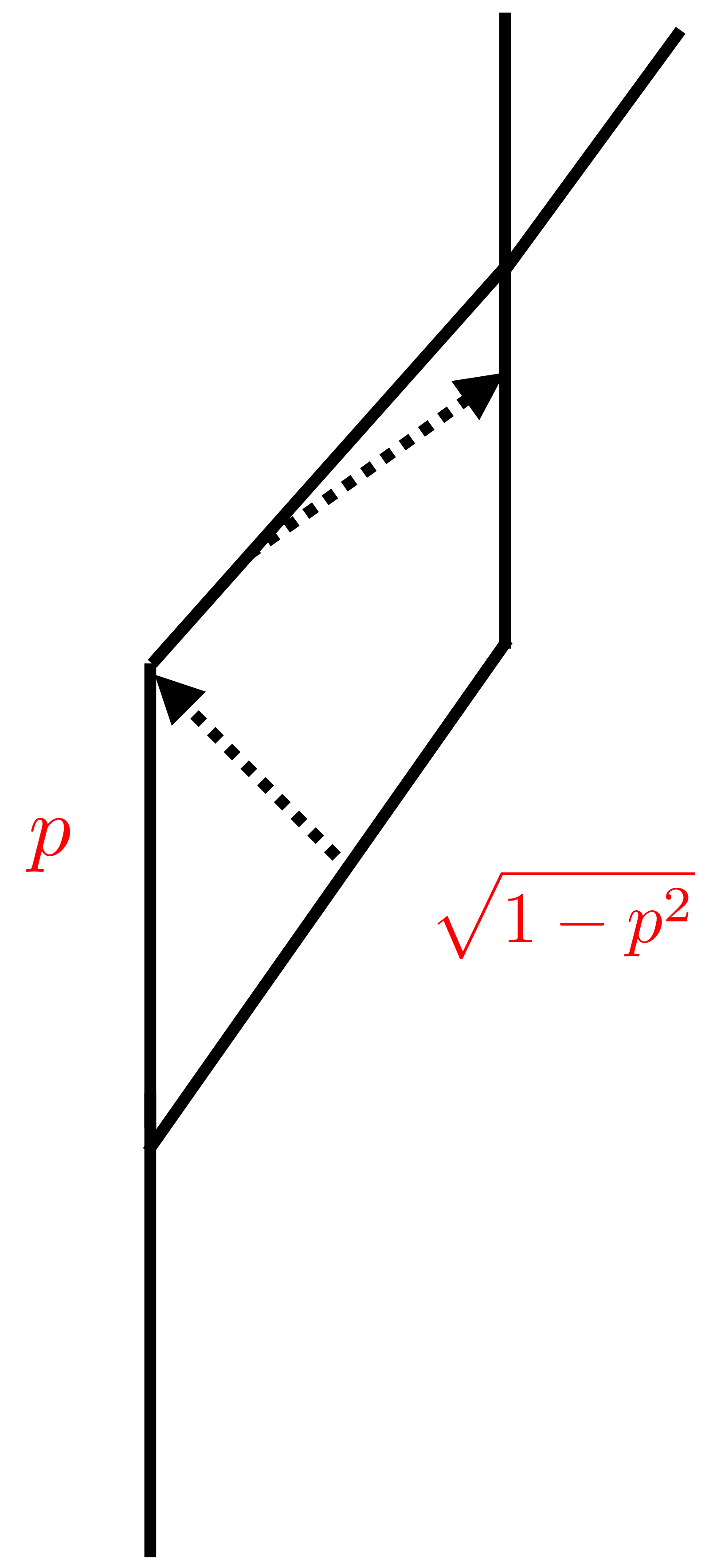
$\varepsilon < 10^{-2}$

Similarly, kills possible bounds on QCD and gravity

Experimental Tests

Interferometry - interaction between paths

Take an ion - split its wave-function

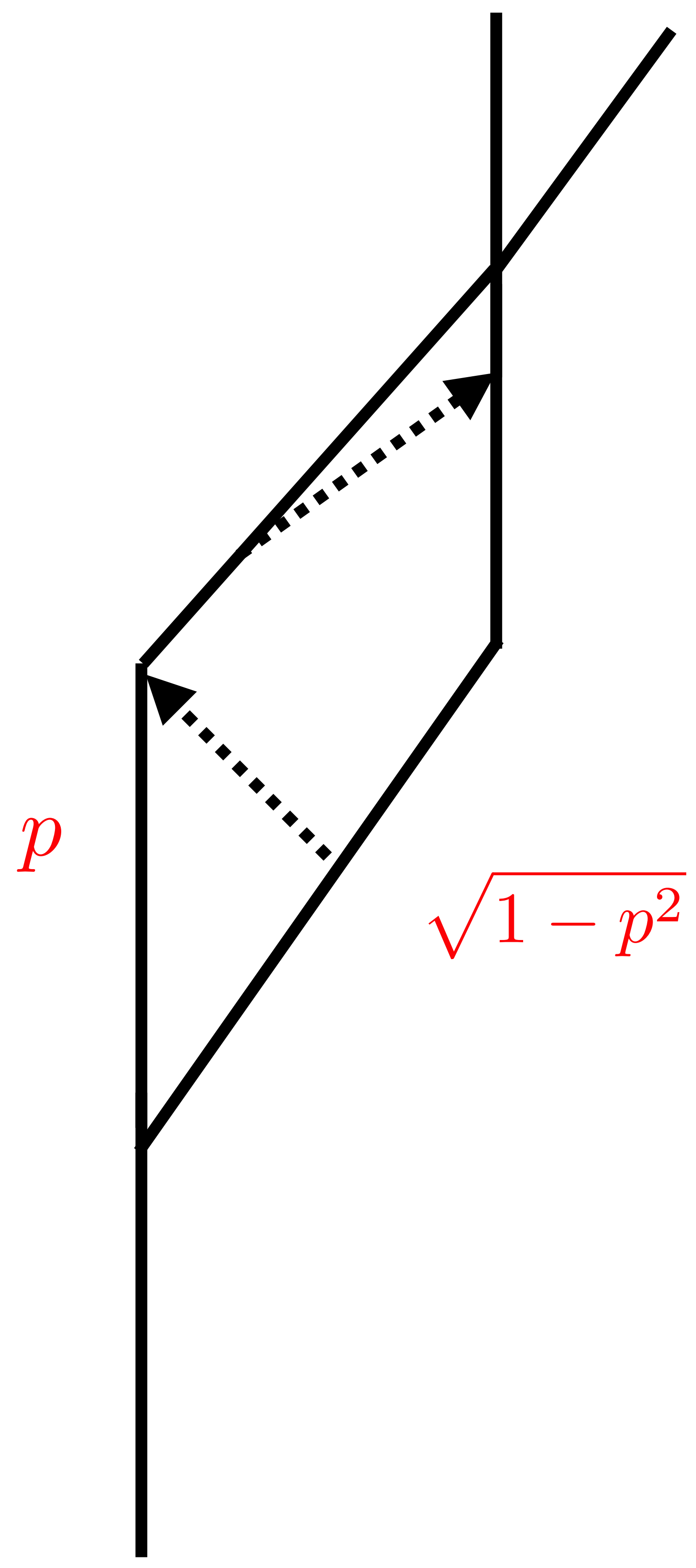


Coulomb Field of one path interacts with the other path

Experimental Tests

Interferometry - interaction between paths

Take an ion - split its wave-function



Coulomb Field of one path interacts with the other path

Gives rise to phase shift that depends on the intensity p of the split

Use intensity dependence to combat systematics

Conclusions

1. Quantum Field Theory can be generalized to include non-linear, state dependent time evolution

2. Conventional tests of quantum mechanics in atomic and nuclear systems do NOT probe causal non-linear quantum mechanics

3. Straightforward set of experimental tests possible to probe non-linear quantum mechanics

4. Motivation to test other extensions as well - e.g. Lindblad Decoherence