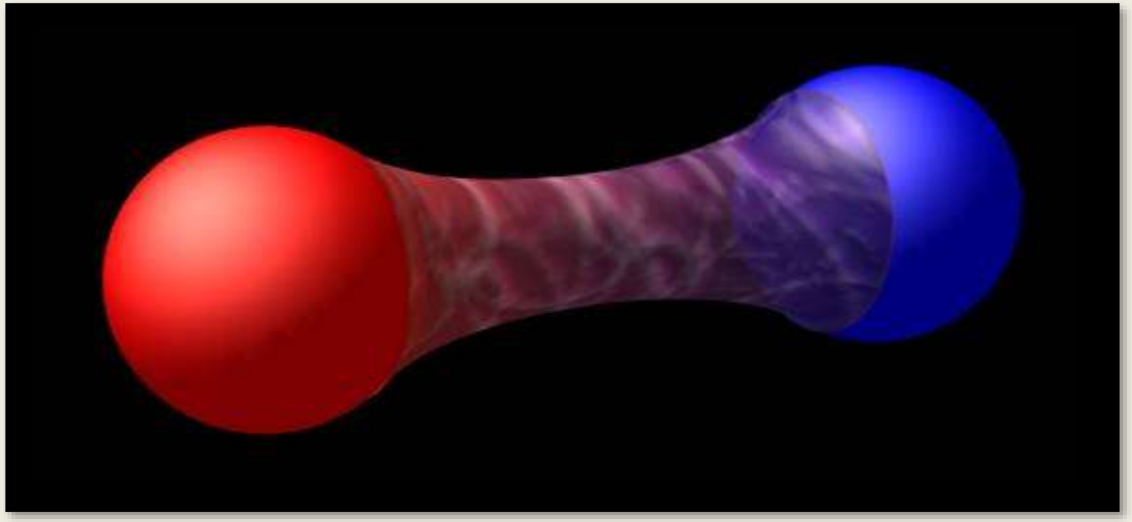
Clocks & Quantum Logic Spectroscopy I



P. O. Schmidt

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PTB Braunschweig and Leibniz Universität Hannover

Frontiers of Quantum Metrology for New Physics Searches

Bad Honnef Physics School, Physikzentrum Bad Honnef, May 11 - 16, 2025

The measurement with the most significant digits ever performed:

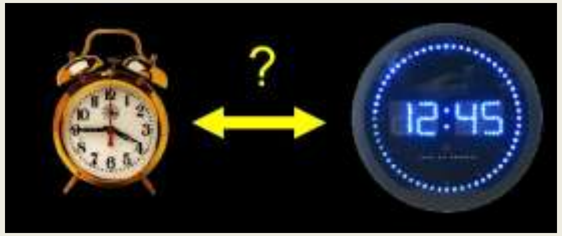
1.973 773 591 557 215 789(9)

(Quantum) Metrology with optical clocks

The measurement with the most significant digits ever performed:

$$\frac{f_{\mathrm{In}^+}}{f_{\mathrm{Yb}^+\,E3}} = 1.973\,773\,591\,557\,215\,789(9)$$

[Hausser *et al.*, PRL **134**, 023201 (2025)]

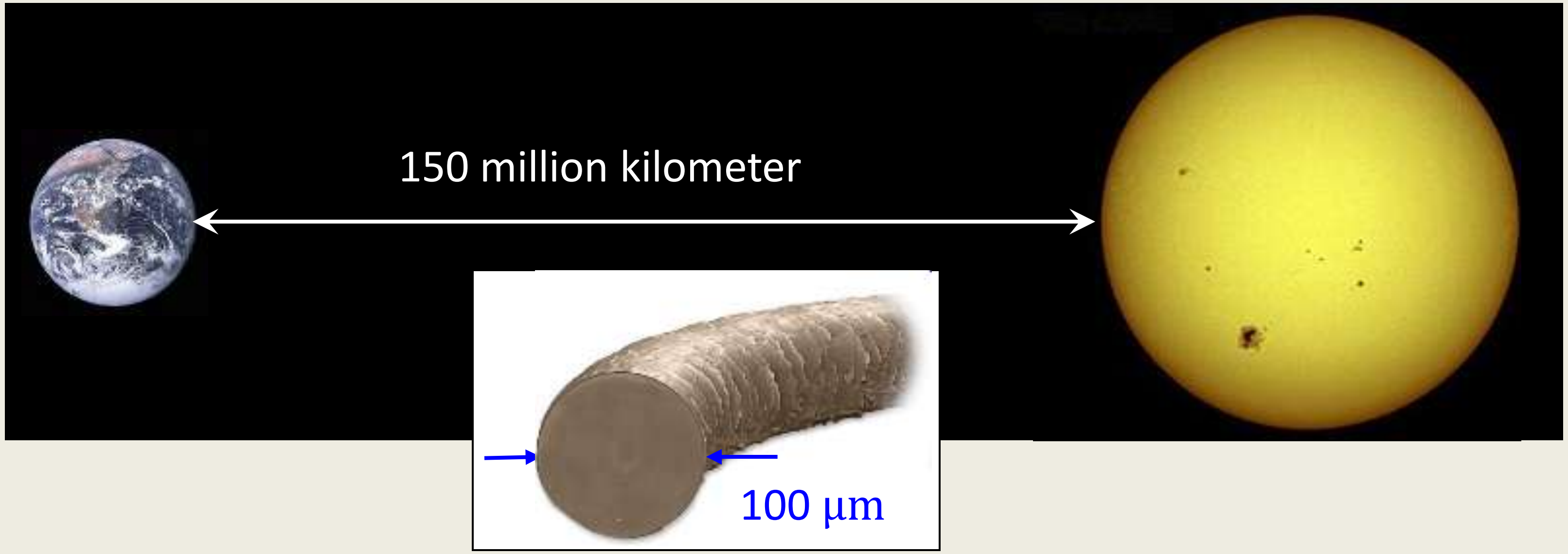


optical clock comparison with 18 digits

…no fundamental limit for improvement in sight (?)

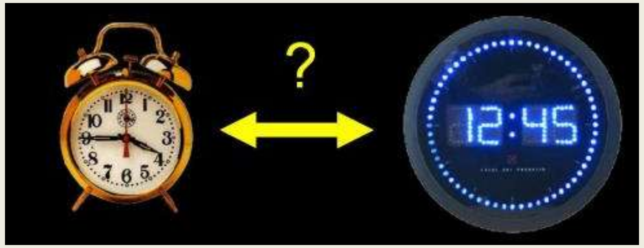
What does 10^{-18} mean?

- 1 : 1,000,000,000,000,000,000
- 300x better than Cs fountain clocks
- 1 s deviation in 30 billion years
- 1st order Doppler shift: 0.3 nm/s or 30 mm/Jahr
- Distance measurement earth-sun to 1/1000 of the diameter of a hair



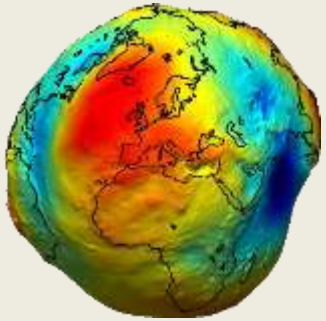
Who Needs Better Clocks?

- Clocks have many applications:
  - Tests of fundamental physics
    - Time variation of constants
    - Tests of general relativity
    - Gravitational wave detection
    - Test local Lorentz invariance
    - …
  - Geodesy
  - Synchronization of large networks
  - Navigation
  - …



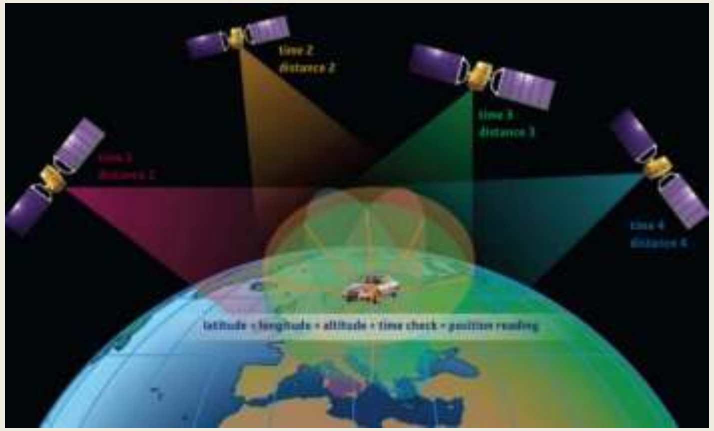
CRC 1227

EXC 2123



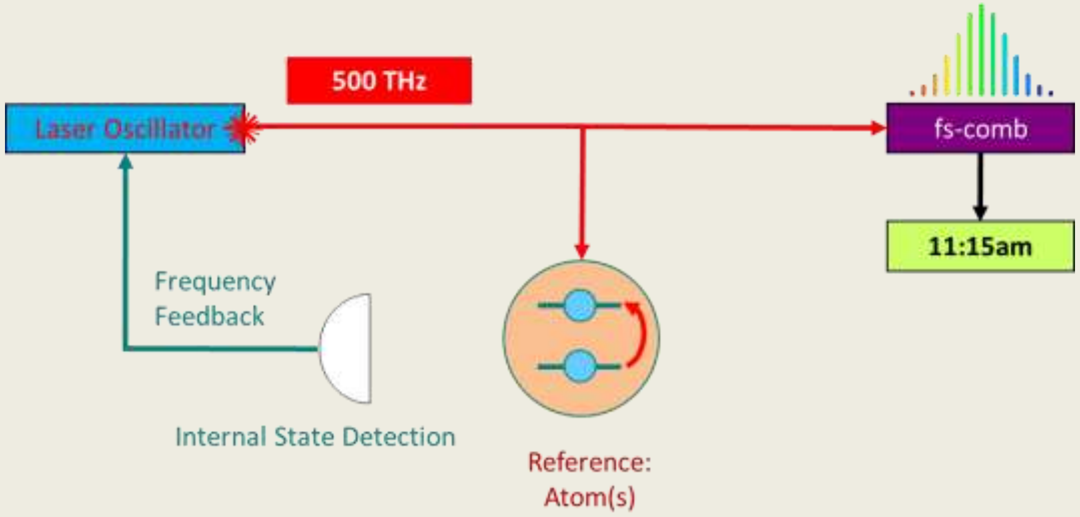
CRC 1464

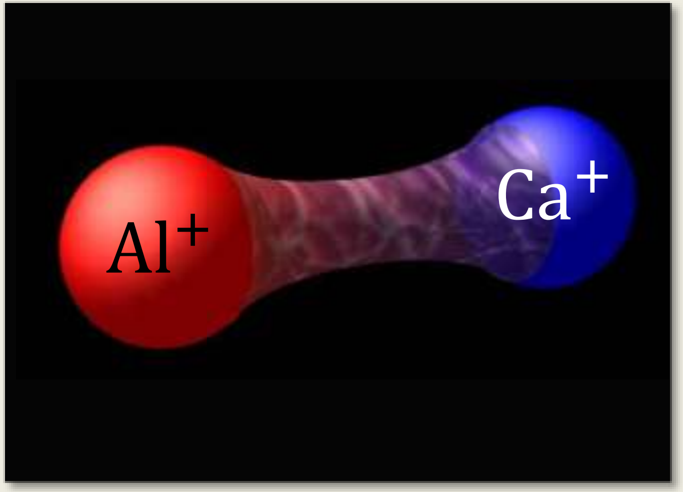
opticlock

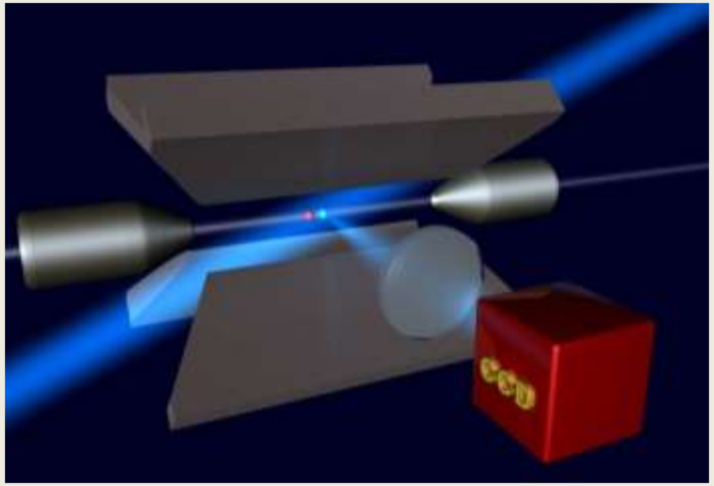


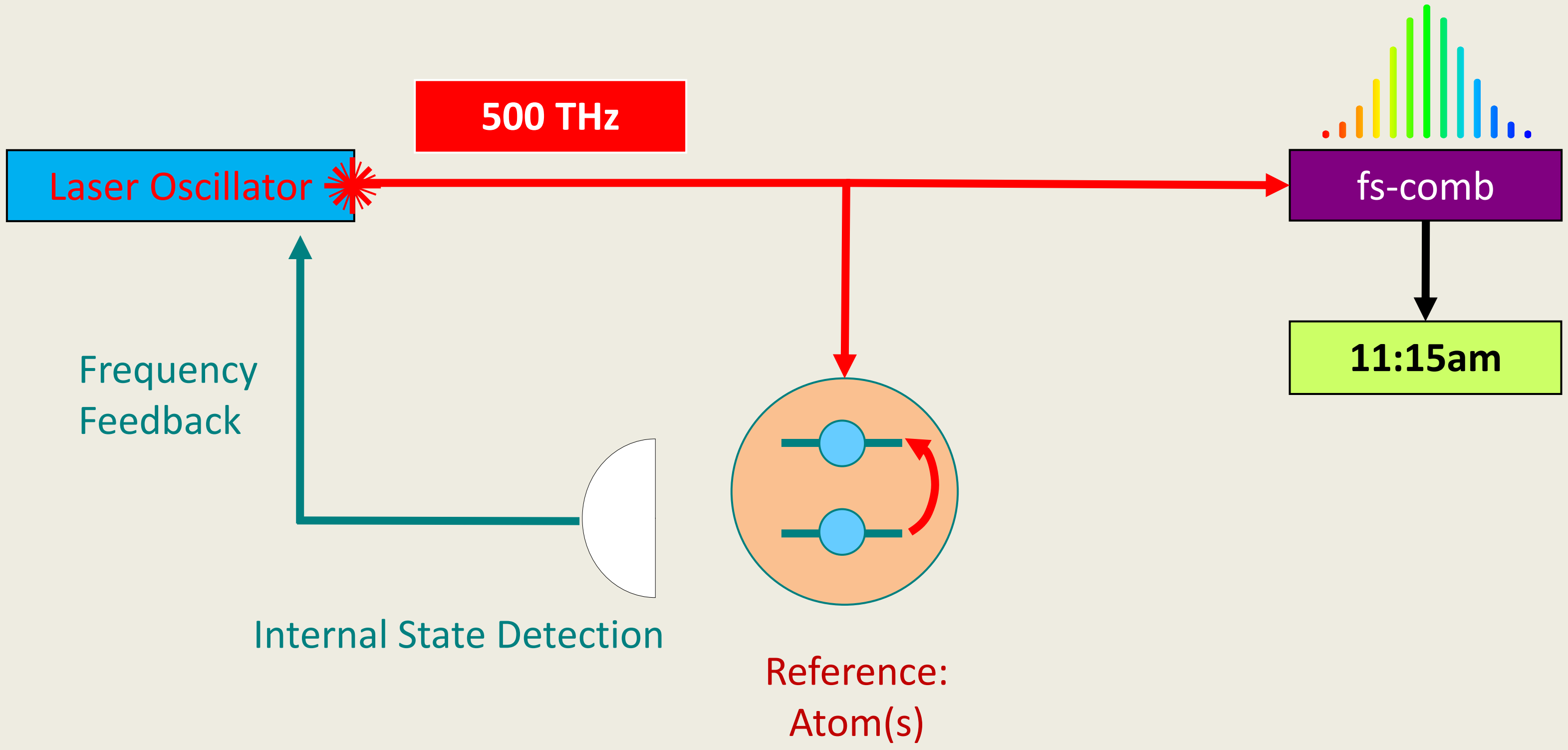
Overview

- Introduction to clocks
  - Systematic & statistical uncertainties
  - Clock cycle
  - Optical frequency comparisons
- Quantum logic spectroscopy I
  - Quantum logic with trapped ions
  - The Al^+ quantum logic clock
- Summary



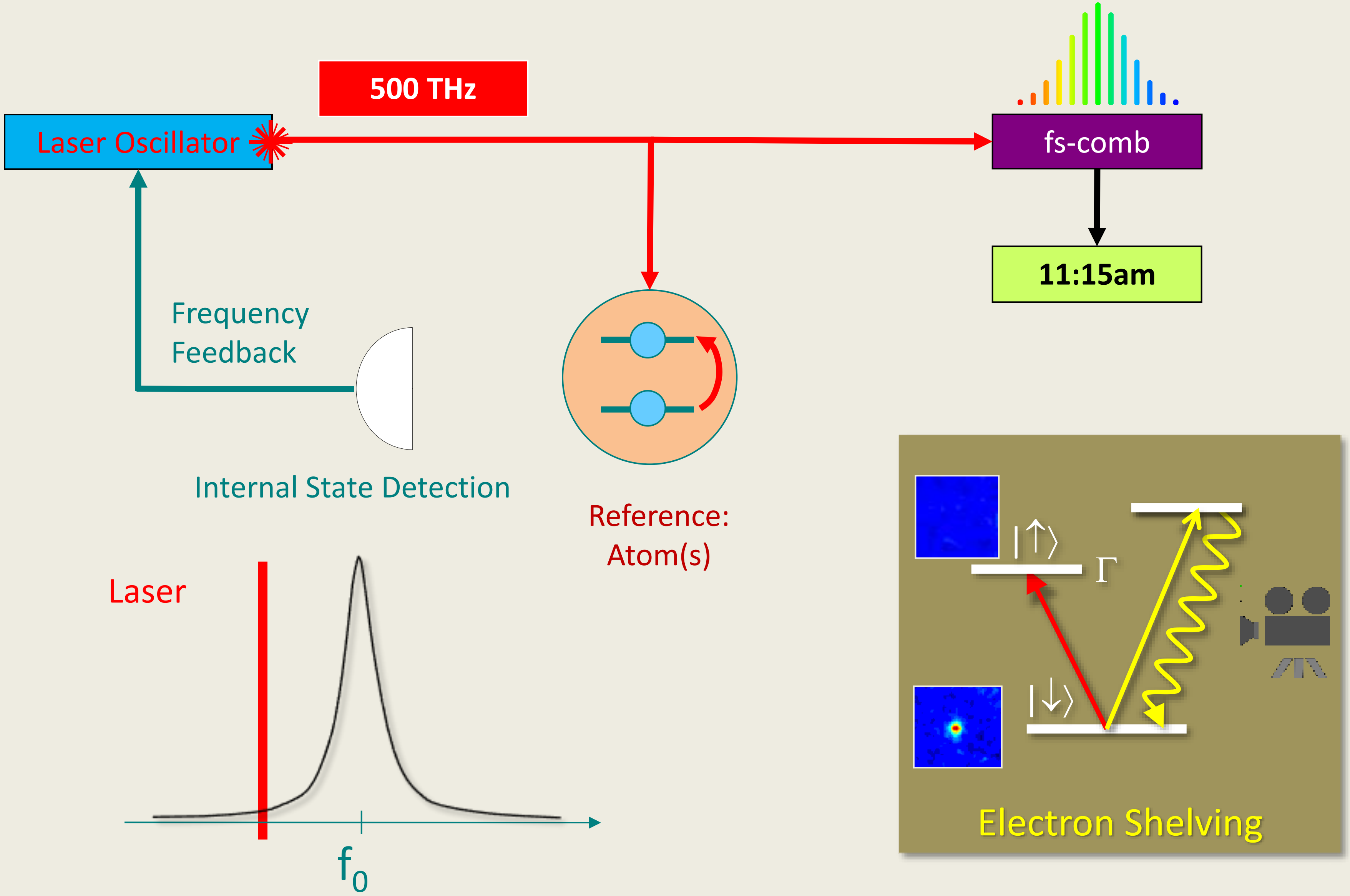




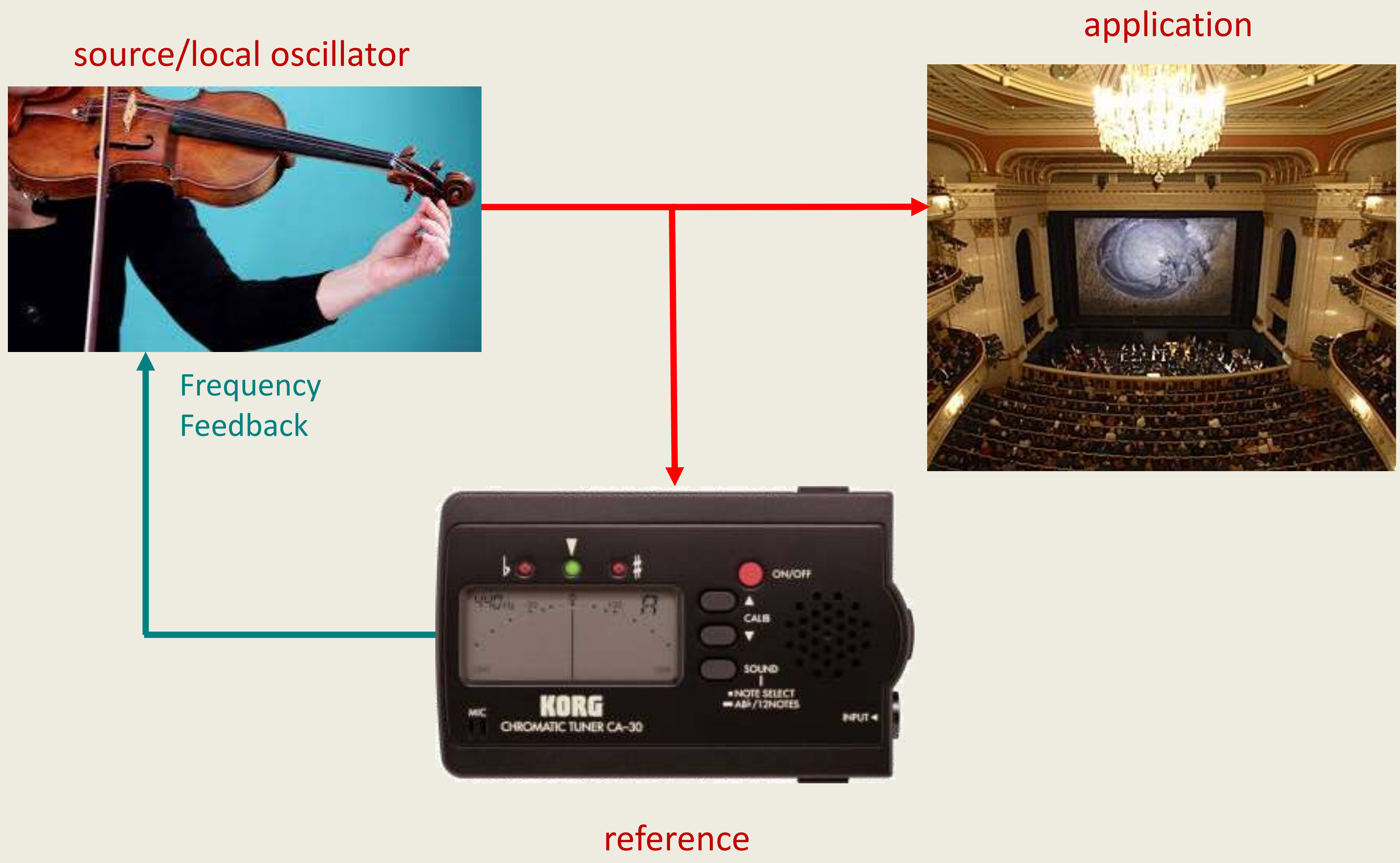


INTRODUCTION TO CLOCKS

Principle of Optical Clocks

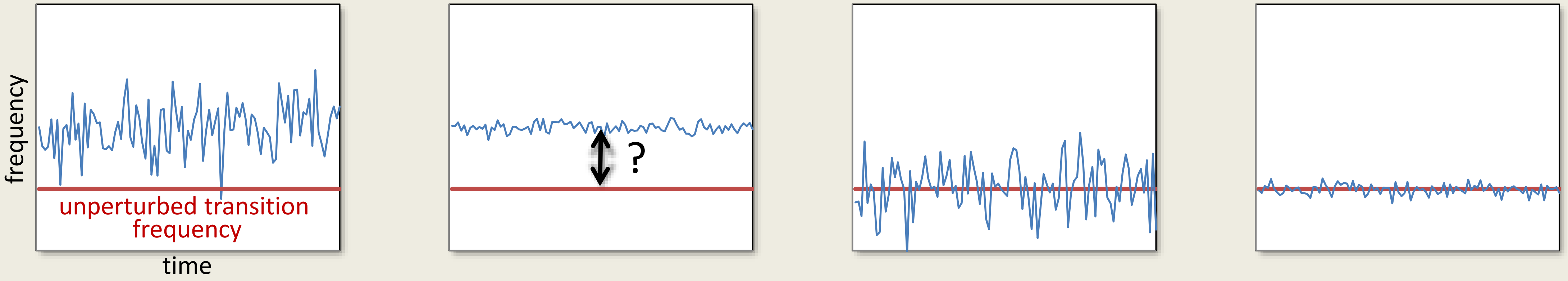


Analogy: Tuning a musical instrument



Clock Characterization

Two (main) notions: Stability & Accuracy



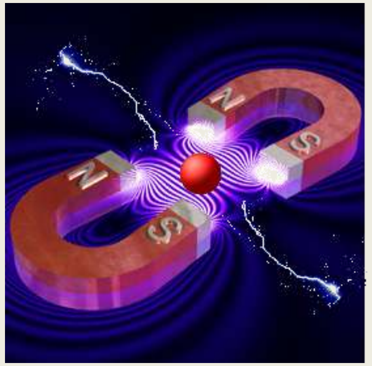
unstable, inaccurate	stable, inaccurate	unstable, accurate	stable, accurate

- (In-)Stability: How large is the scatter of data points?
  ➔ Uncertainty type A (u_A, only statistical) "statistical uncertainty"
- (In-)Accuracy: How well is the unperturbed reference reproduced?
  ➔ Uncertainty type B (u_B, outside information) "systematic uncertainty"

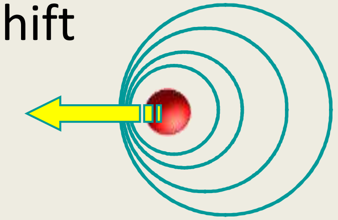
Problem: value of unperturbed reference a priori not known!

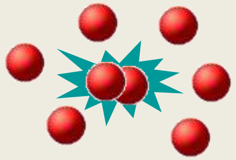
Typical systematic shifts

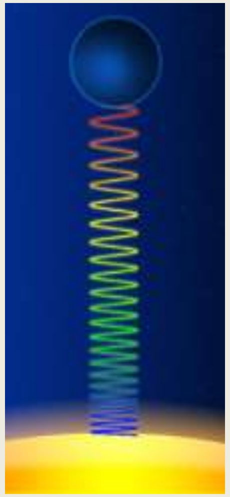
- electric & magnetic fields
  - black-body radiation shift
  - Stark shifts
  - electric quadrupole shift
  - Zeeman shifts
  - lattice light shift
- …



- relativistic shifts
  - time dilation (2nd order Doppler)
  - gravitational redshift
- collision shifts







- evaluation of frequency shifts and systematic uncertainties:
  - usually 2 contributions: e.g. $\mathrm{u_B}\left(\frac{\Delta f_{BBR}}{f}\right) = \sqrt{\left(\frac{\delta\alpha_s}{\alpha_s}\right)^2 + 2\left(\frac{\delta E}{E}\right)^2}$

atomic property

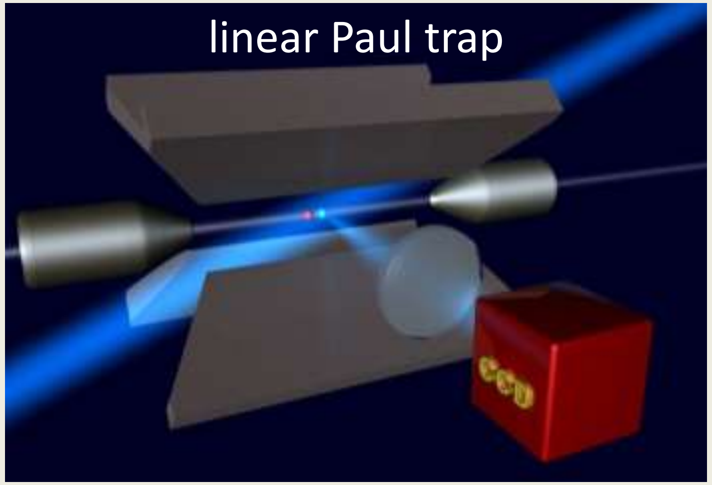
external field

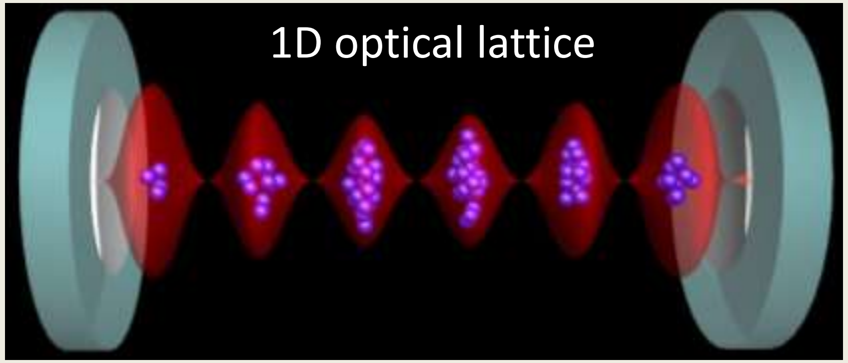
[Ludlow, Boyd, Ye, Peik, Schmidt, Rev. Mod. Phys. **87**, 637 (2015)]

Quantum control over all degrees of freedom

Quantum Technology 2.0

- localisation & motional control → trapping & laser cooling the atoms
- internal state control → optical, radiofrequency or microwave interaction



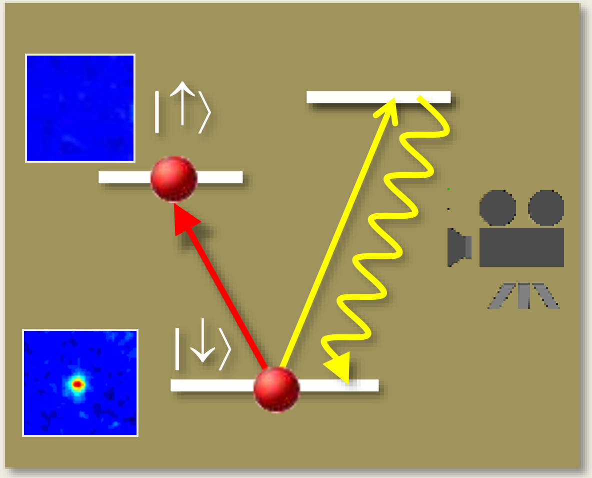


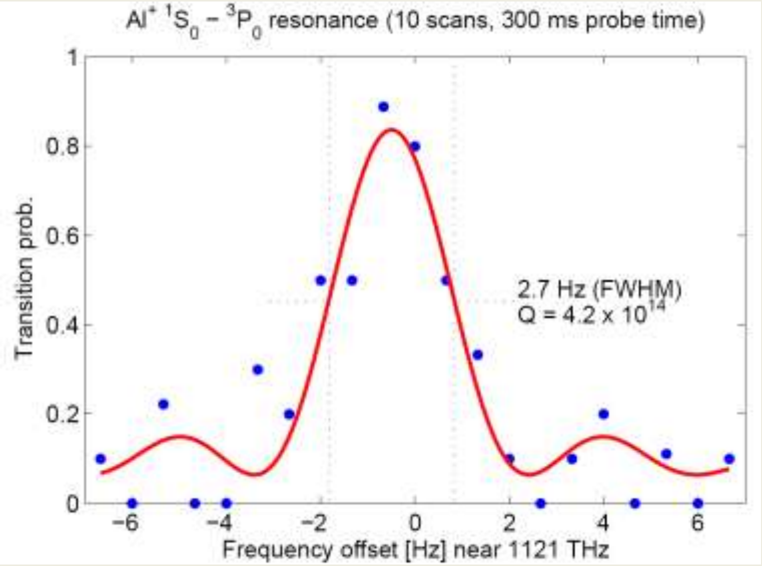
advantage:
- few and small systematic shifts

advantage:
- small statistical uncertainty (1000s of atoms)

Rabi spectroscopy

- prepare atom in $|\downarrow\rangle$
- apply spectroscopy laser pulse
  $|\downarrow\rangle \rightarrow \alpha|\downarrow\rangle + \beta|\uparrow\rangle$
- detect via fast cycling transition
  ➔ $|\downarrow\rangle$ (bright) or $|\uparrow\rangle$ (dark)
  with probabilities $|\alpha|^2, |\beta|^2$
- repeat many times or use many atoms
- change spectroscopy laser frequency
  ➔ lock laser to resonance line
- long probe time → narrow linewidth Δf
  (limited by natural linewidth)

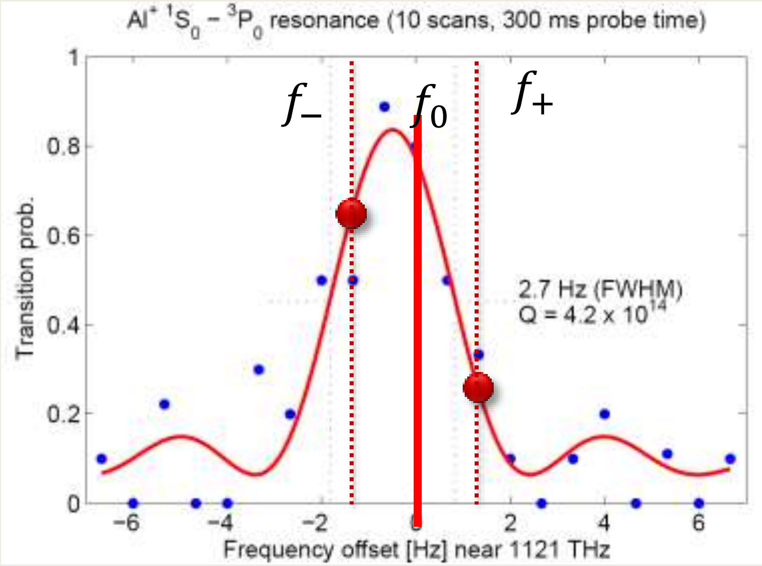




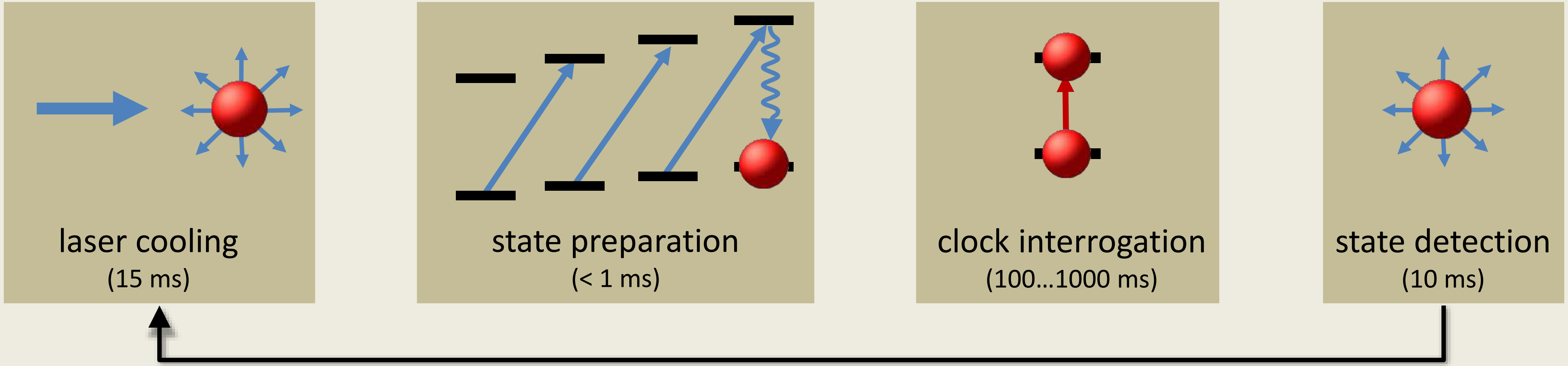
[C.W. Chou, *et al.* Science **329**, 1630 (2010)]

Feedback loop

- Interrogate line at $f_{\pm} = f_0 \pm \delta_m$
- δ_m is chosen to achieve $\mathrm{p}_{\pm} \approx 0.5$
- error signal: $\epsilon = \delta_m(p_+ - p_-)$
- add correction to laser frequency (PI[2])
  → steer laser towards f_0
- full clock sequence:



[C.W. Chou, *et al.* Science **329**, 1630 (2010)]

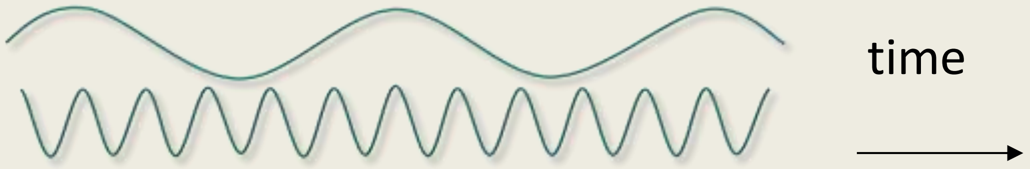


[Ludlow, Boyd, Ye, Peik, Schmidt, Rev. Mod. Phys. **87**, 637 (2015)]

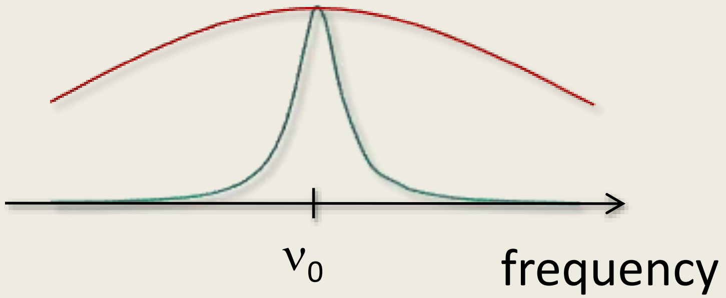
Wishlist for small statistical uncertainty

- High transition frequency

low frequency (microwave)
$\downarrow \times 10^5$
high frequency (optical)

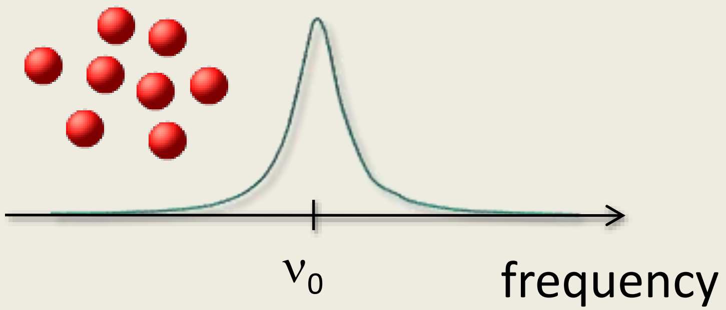


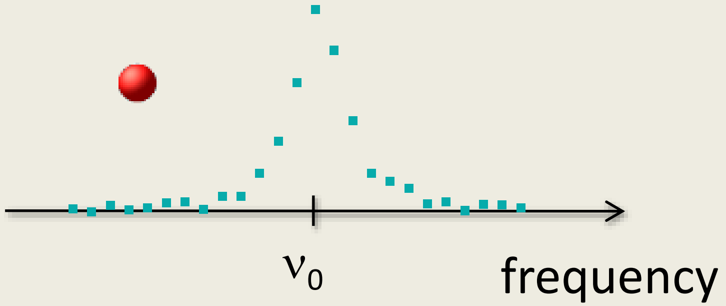
- Long Ramsey time T_R
  → small linewidth $\Delta\nu$



relevant quantity: $\frac{\Delta f}{f_0}$

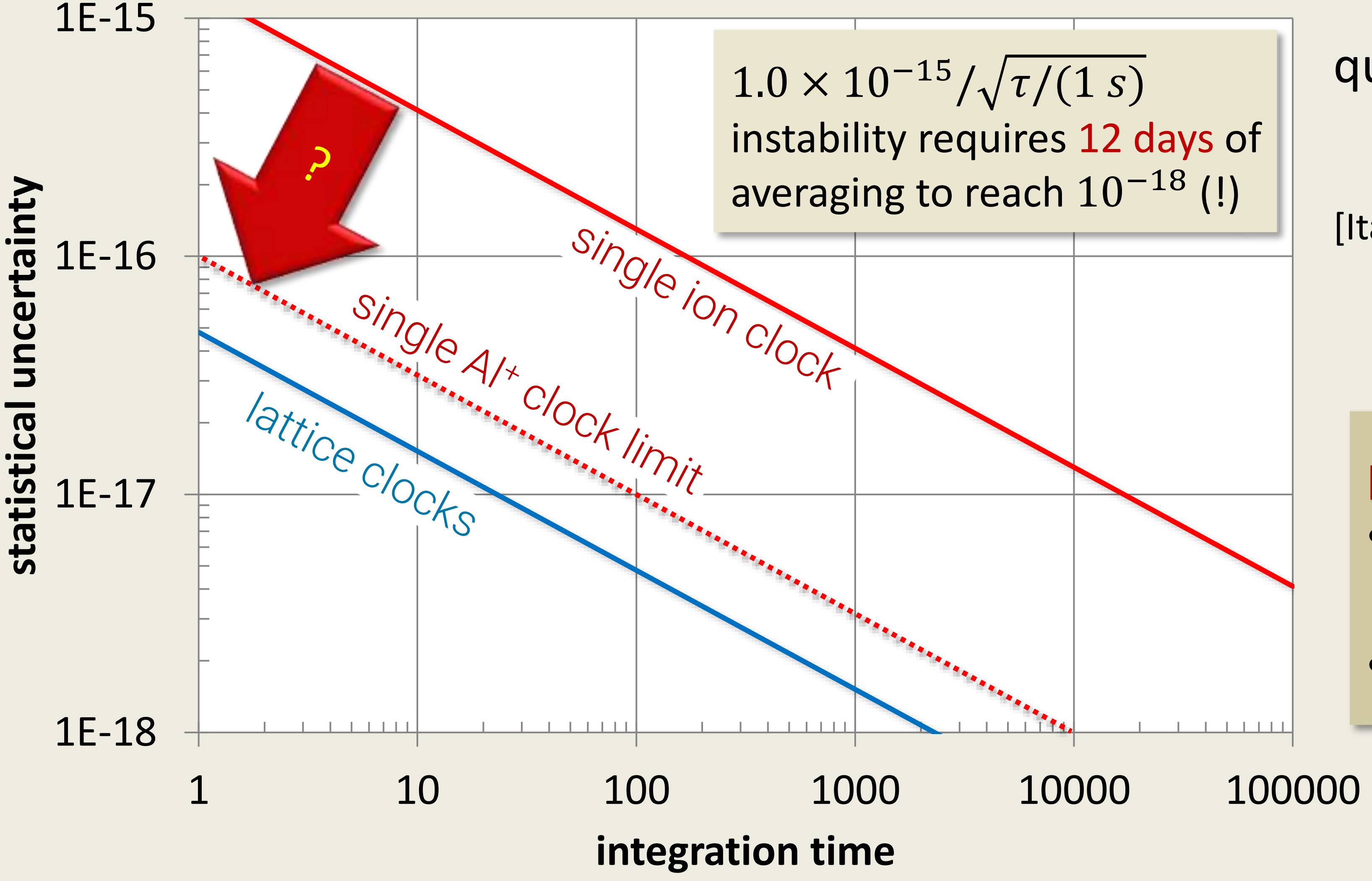
- large number of atoms





- Quantum Projection Noise: $\sigma_y(\tau) = \frac{1}{2\pi f_0 \sqrt{N T_R \tau}}$

State of the art: stability



quantum projection noise limit:

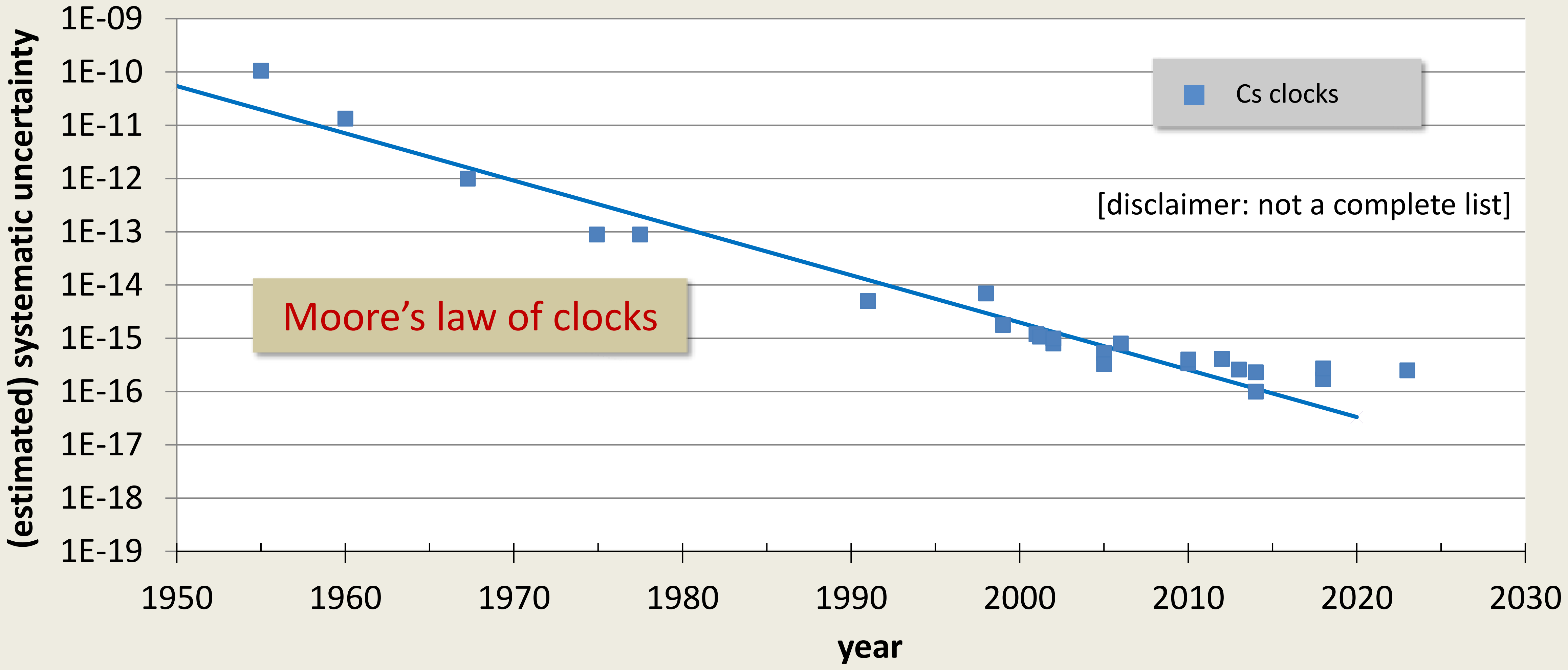
$$\sigma_y(\tau) \approx \frac{1}{\omega_0\sqrt{N T_R \tau}}$$

[Itano *et al.*, PRA **47**, 3554–3570 (1993)]

probe time T_R limited by

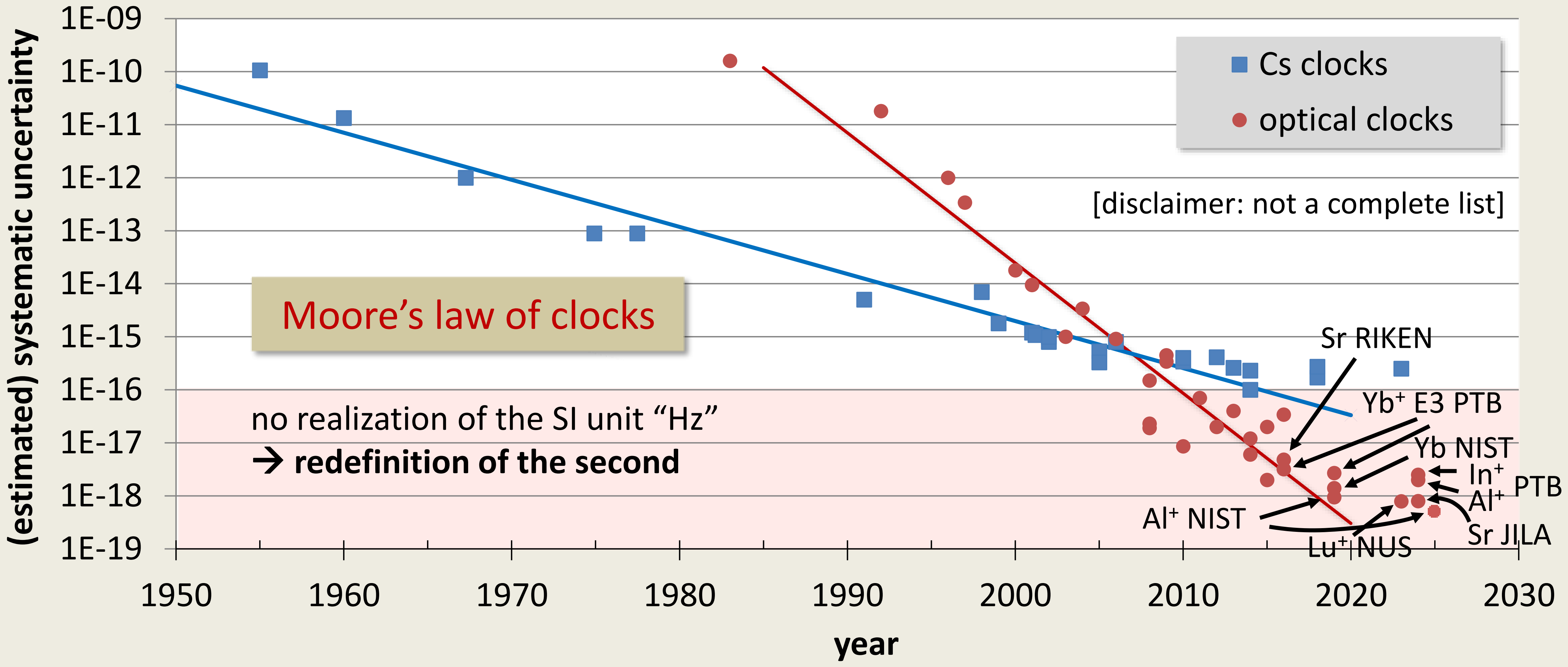
- excited state lifetime (natural linewidth)
- laser coherence time

Evolution of (estimated) clock accuracy



Definition of the second (since 1967): *One Second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.*

Evolution of (estimated) clock accuracy

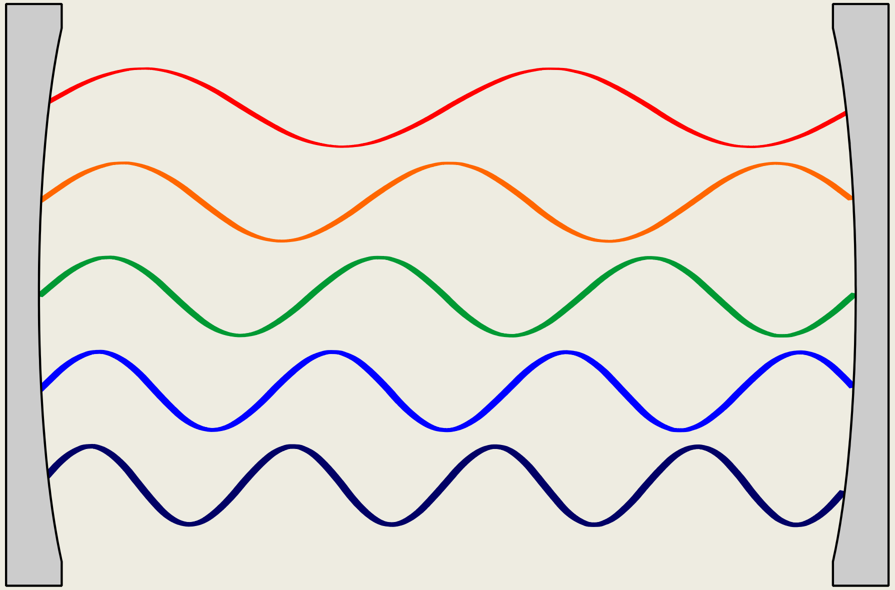


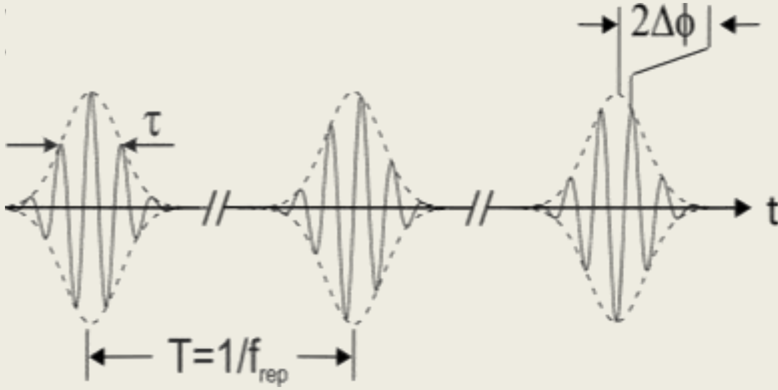
Definition of the second (since 1967): *One Second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.*

FREQUENCY COMPARISONS

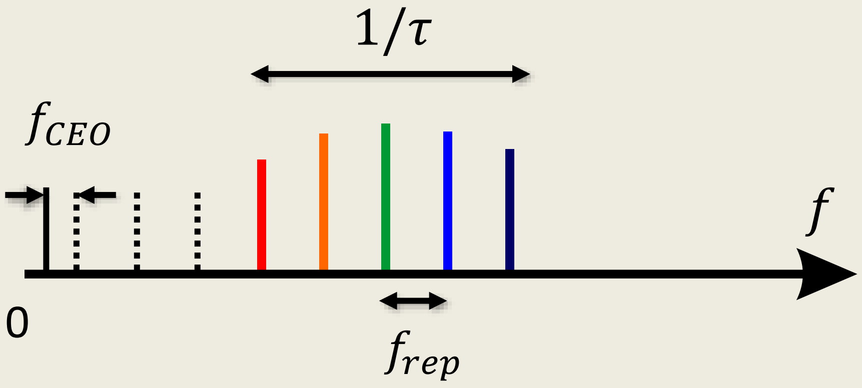
Breakthrough: fs frequency combs

mode-locked pulsed laser:





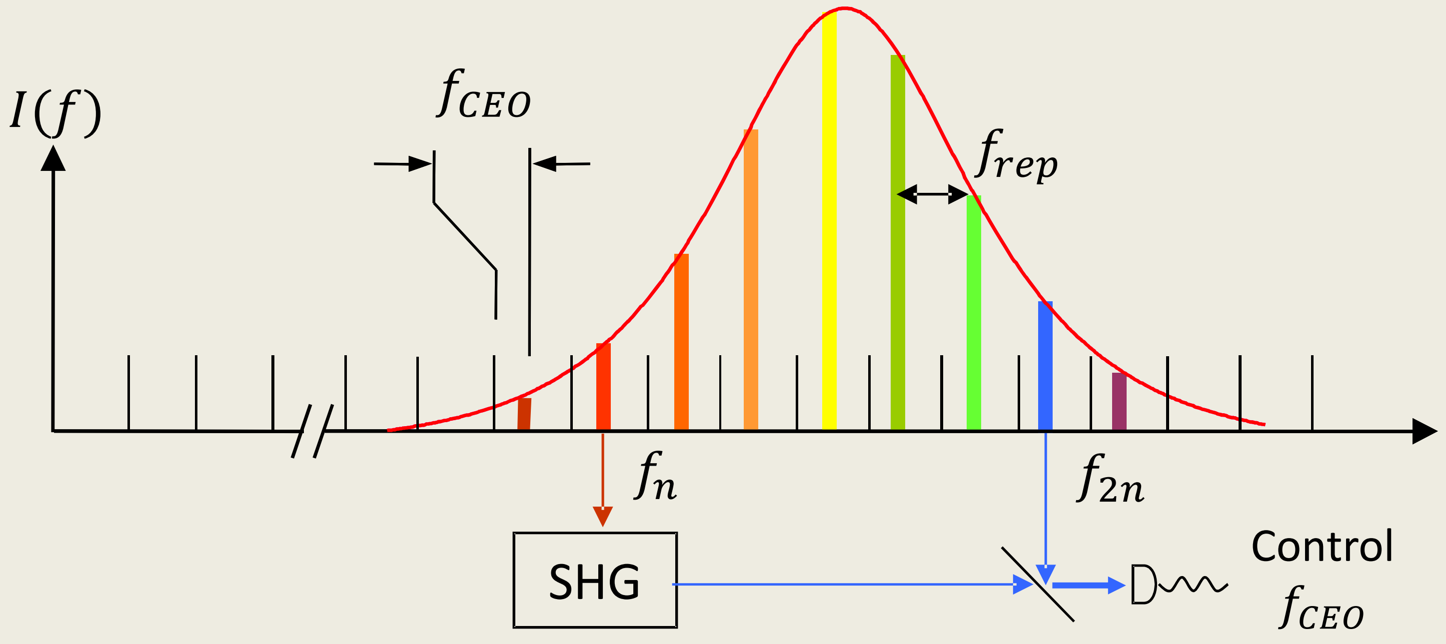
- time regime: fs pulses with pulse width τ & repetition rate $f_{rep} = 1/T_{rep}$
- bandwidth of spectrum: $1/\tau$
- Origin of f_{CEO}: difference between phase and group velocity: $f_{CEO} = \Delta\phi f_{rep}/2\pi$



[Rev. Mod. Phys. **78**, 1279 &1297 (2006)]

Breakthrough: fs frequency combs

mode-locked pulsed laser:



- Comb equation:

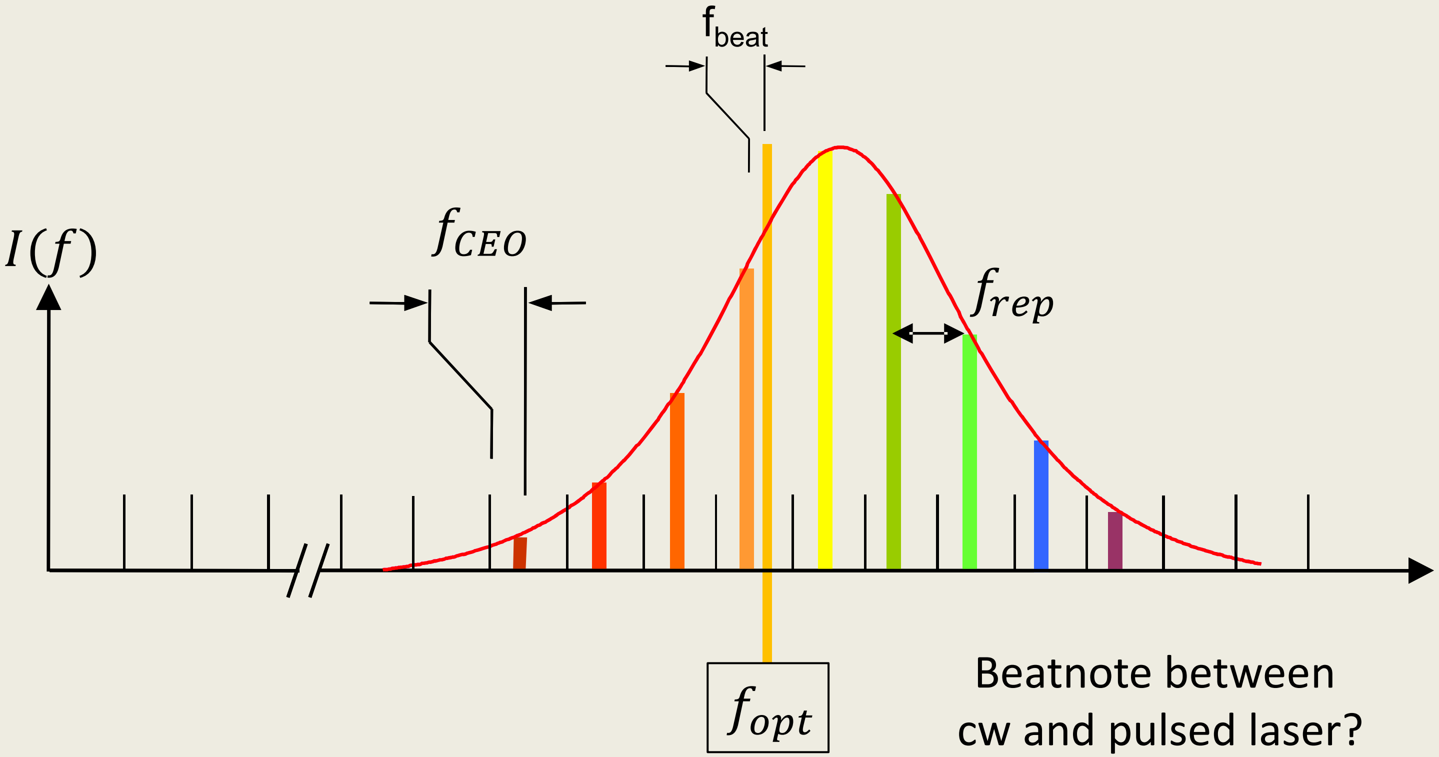
  $f_n = nf_{rep} + f_{CEO}$

- How can we measure f_{CEO}?

  → $f - 2f$ intererometer

Breakthrough: fs frequency combs

mode-locked pulsed laser:



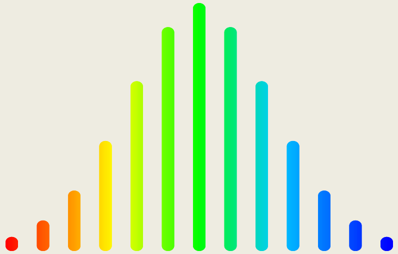
- Comb equation:
  $f_n = nf_{rep} + f_{CEO}$
- Optical frequency:
  $f_{opt} = nf_{rep} + f_{CEO} \pm f_{beat}$

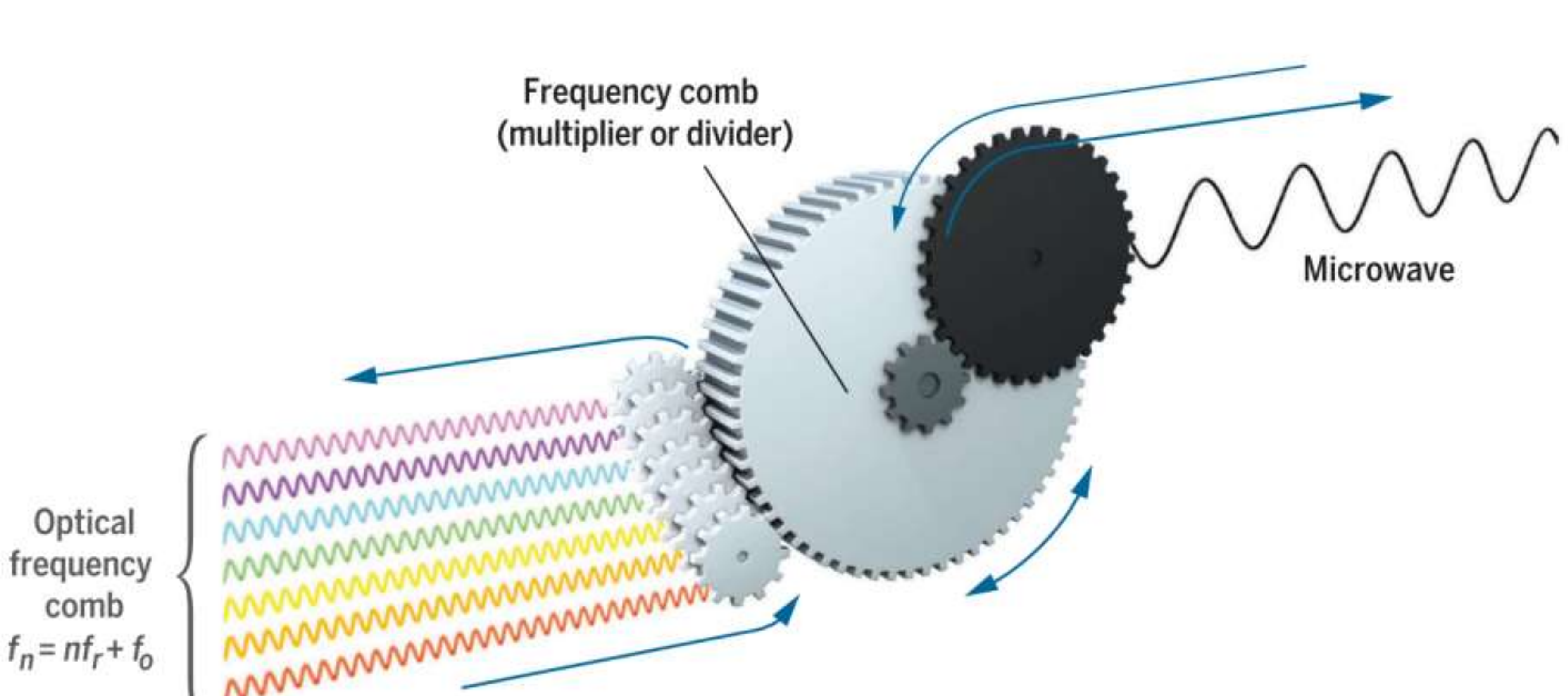
$$f_{opt} = nf_{rep} + f_{CEO} + f_{beat}$$

Reduce optical frequency measurements to rf frequency measurements

Clock comparisons

RF to optical clockwork with a femtosecond laser frequency comb



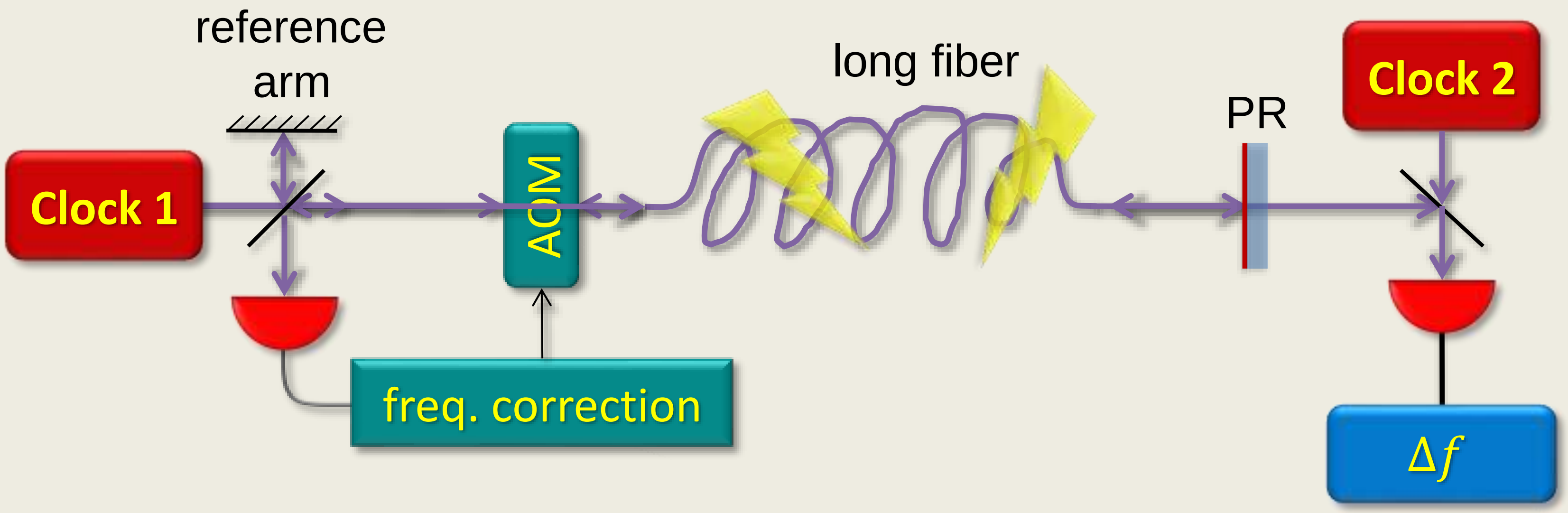


[S. A. Diddams *et. al.*, Science **369**, eaay3676 (2020)]

➔ comparison of optical/rf clocks

Clock Comparison: Phase-Stabilized Fibers

- Acoustic and thermal perturbations affect length of fiber
- frequency of transmitted light is Doppler-shifted
  ➔ active length stabilization of fiber required

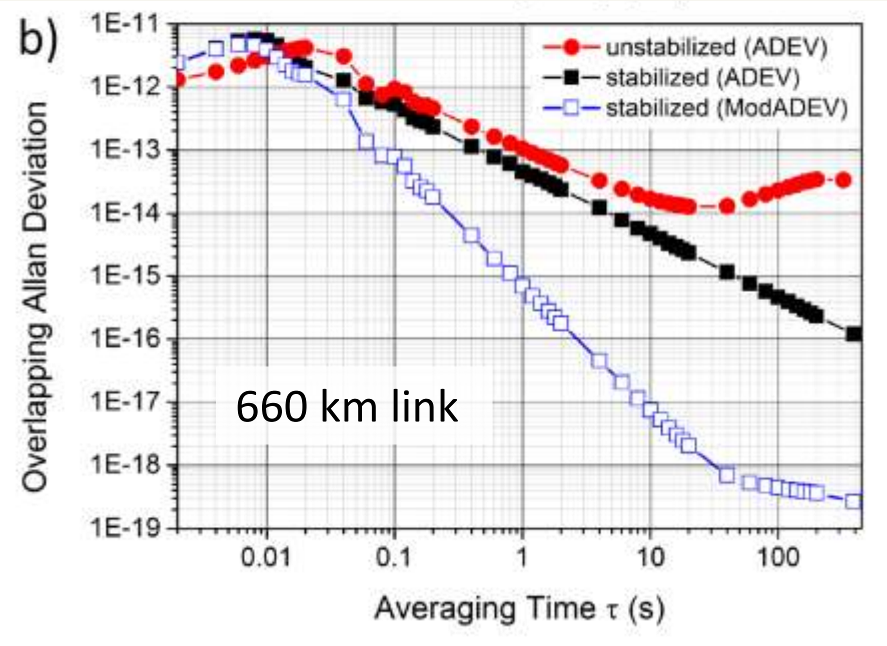


- Works in the lab & for long distances

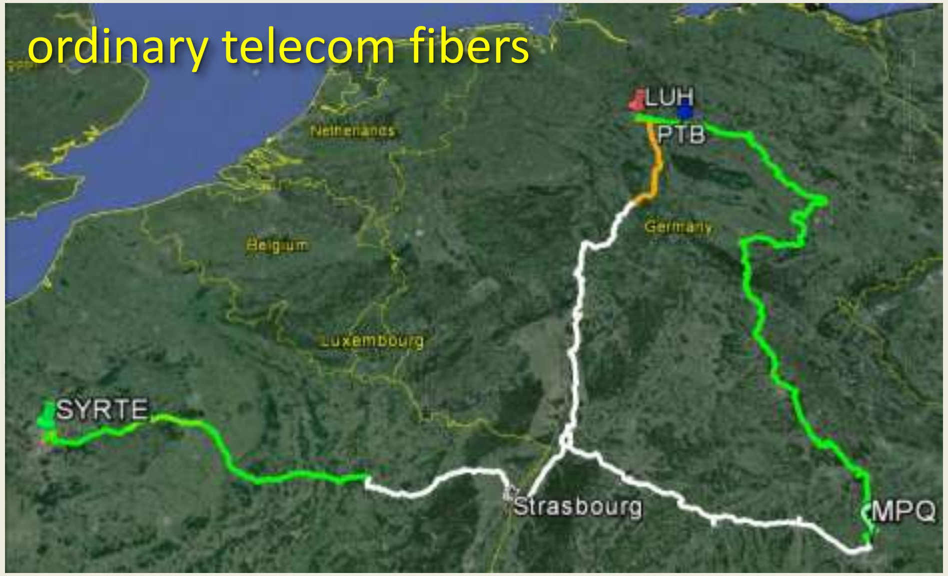
[L.-S. Ma et al., Opt. Lett. **19**, 1777 (1994)]

Performance Fiber Links

- fundamental stability limit: time delay τ in feedback
  ➔ residual PSD: $S_D(f) \approx (2\pi f \tau)^3 S_{fiber}(f)/3$
- non-reciprocal noise
- 3 bidirectional Brillouin amplifiers to compensate 300 dB (!) loss PTB-Strasbourg-PTB



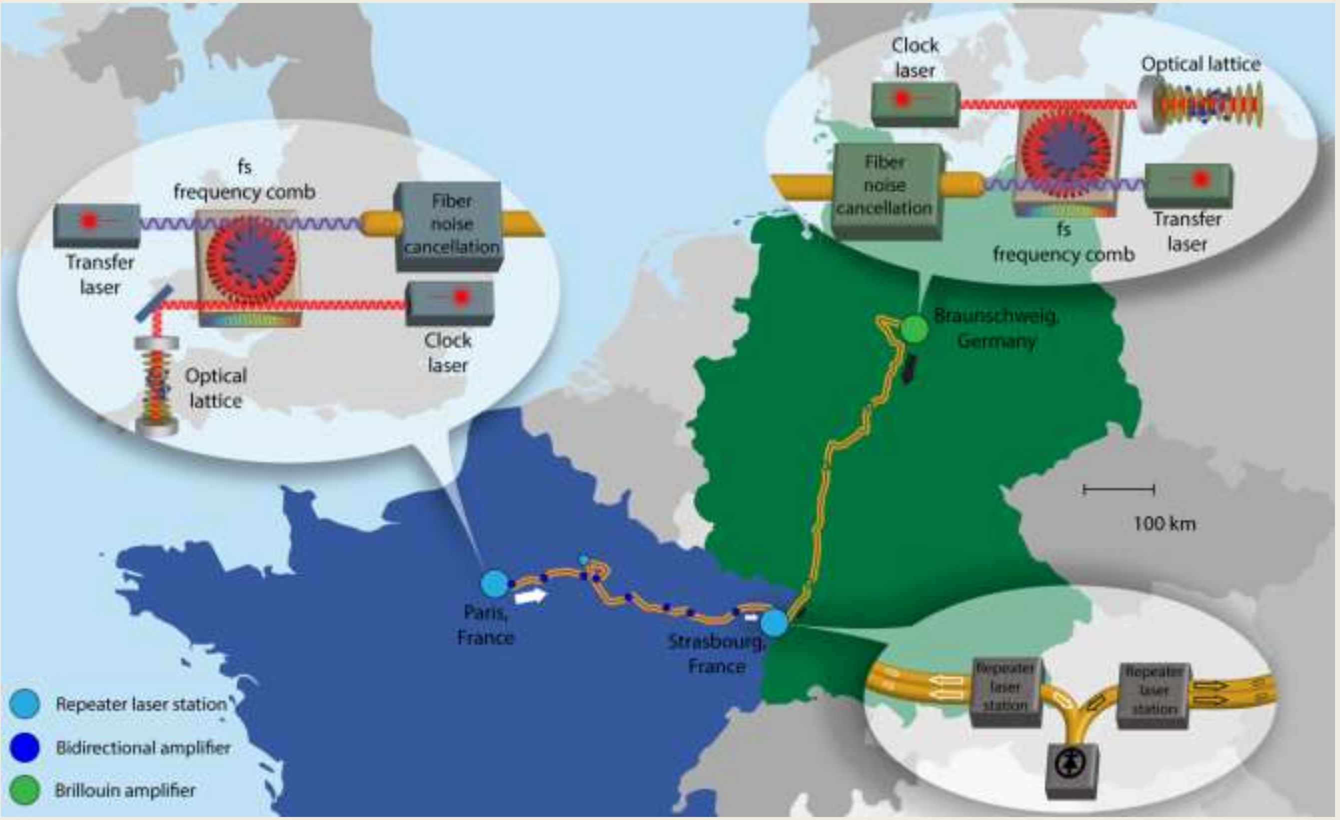
[Raupach *et al.*, Opt. Express **22**, 26537–26547 (2014)]



Group of G. Grosche/H. Schnatz @ PTB

Optical frequency dissemination

Optical fibre-based dissemination



- fibre required

→ telecom, 1.5 µm

- bidirectional for noise cancellation

→ incompatible with telecom

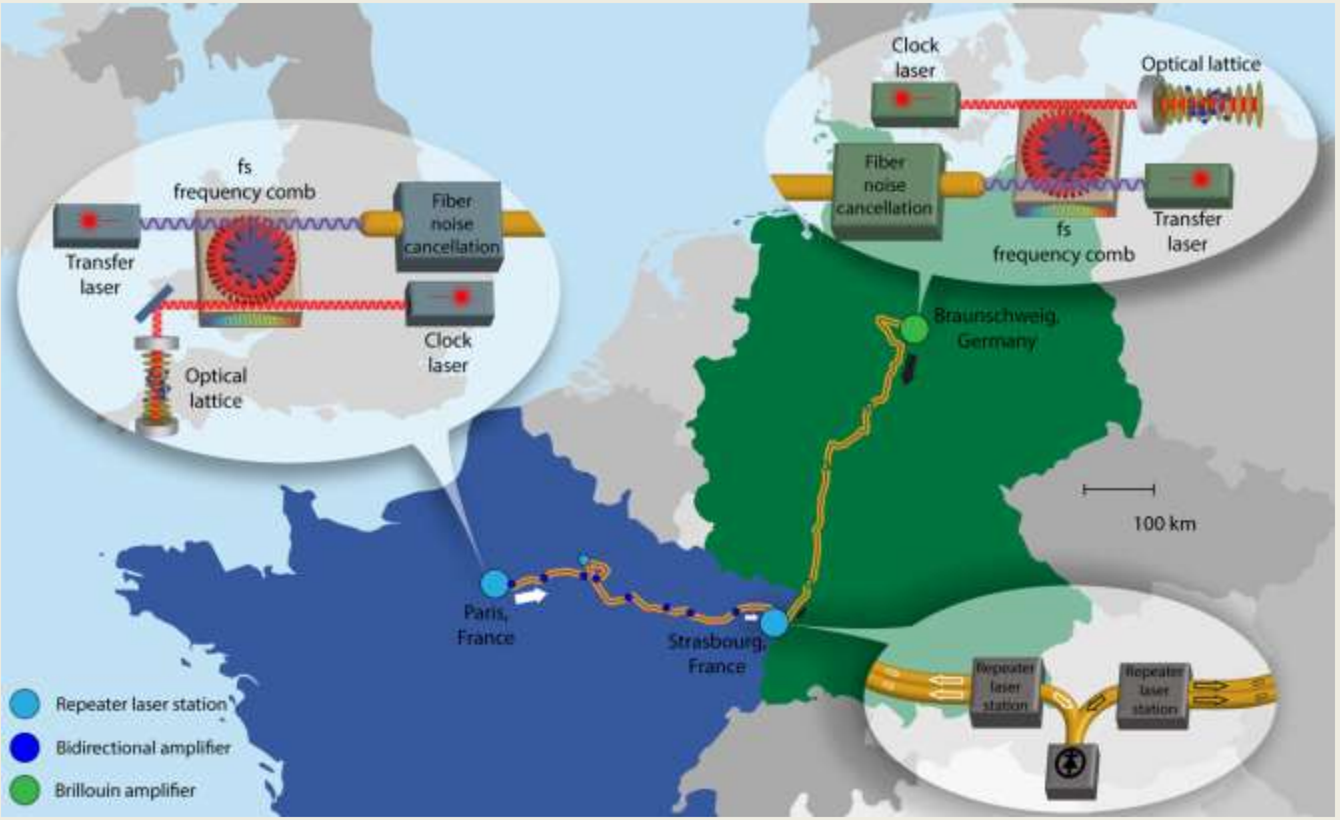
- 200 dB loss per link

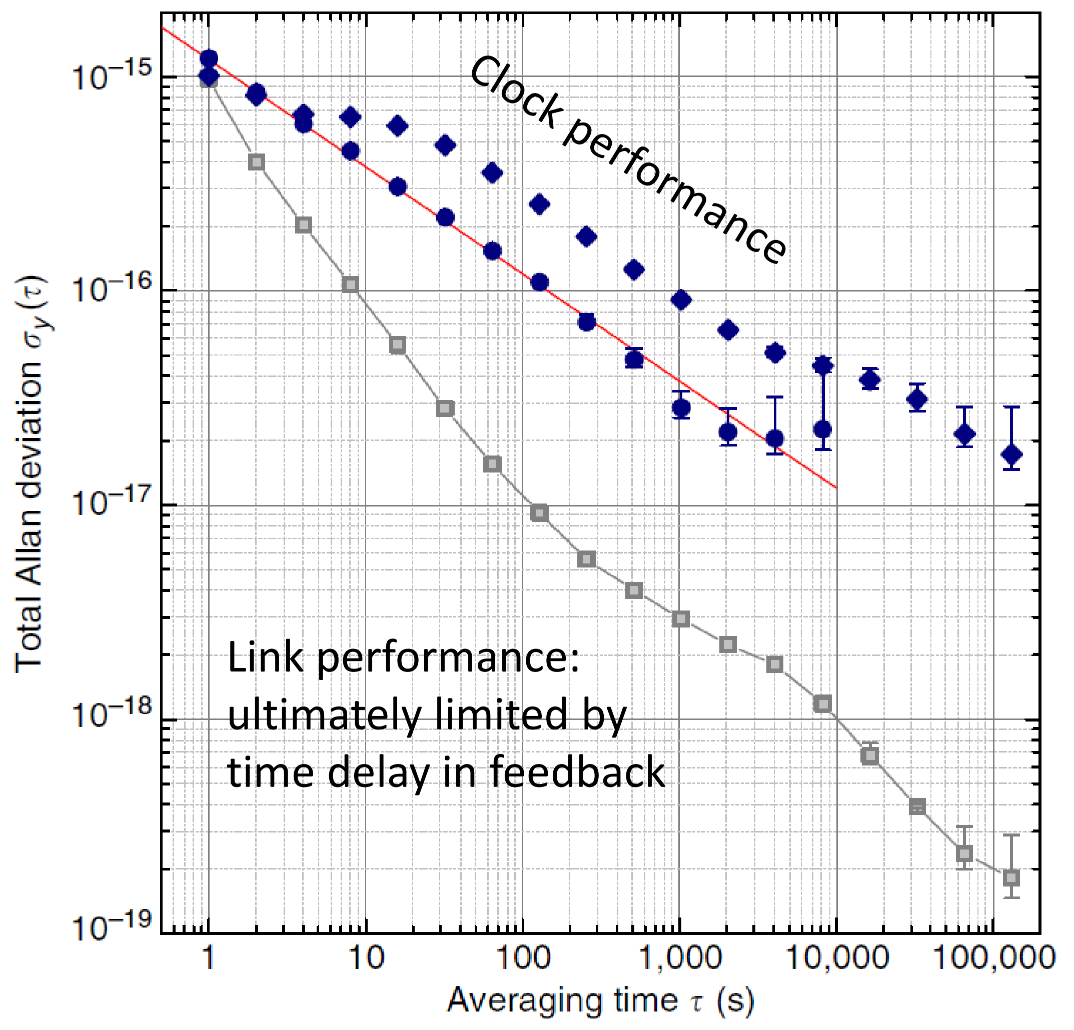
→amplifiers required, usually unidirectional

[Ch. Lisdat *et al*, Nature Comm. **7**, 12443 (2016)]

Optical frequency dissemination

Connecting clocks in Europe



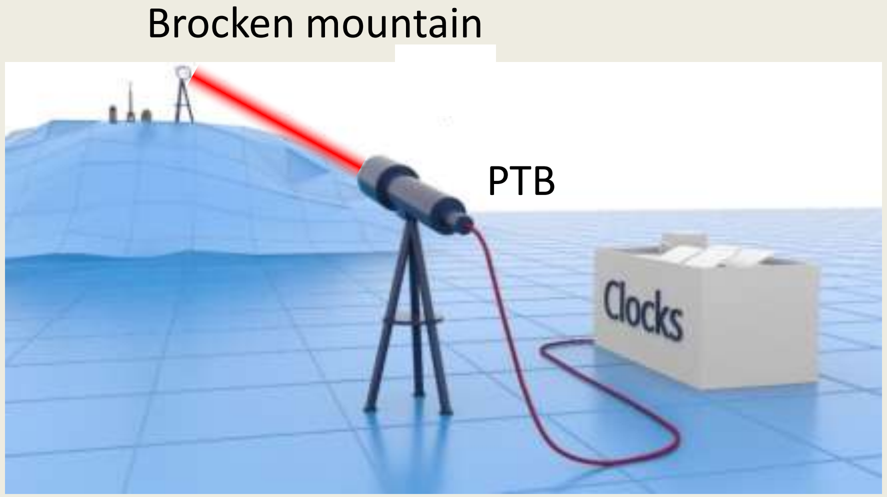


[Ch. Lisdat *et al*, Nature Comm. **7**, 12443 (2016)]

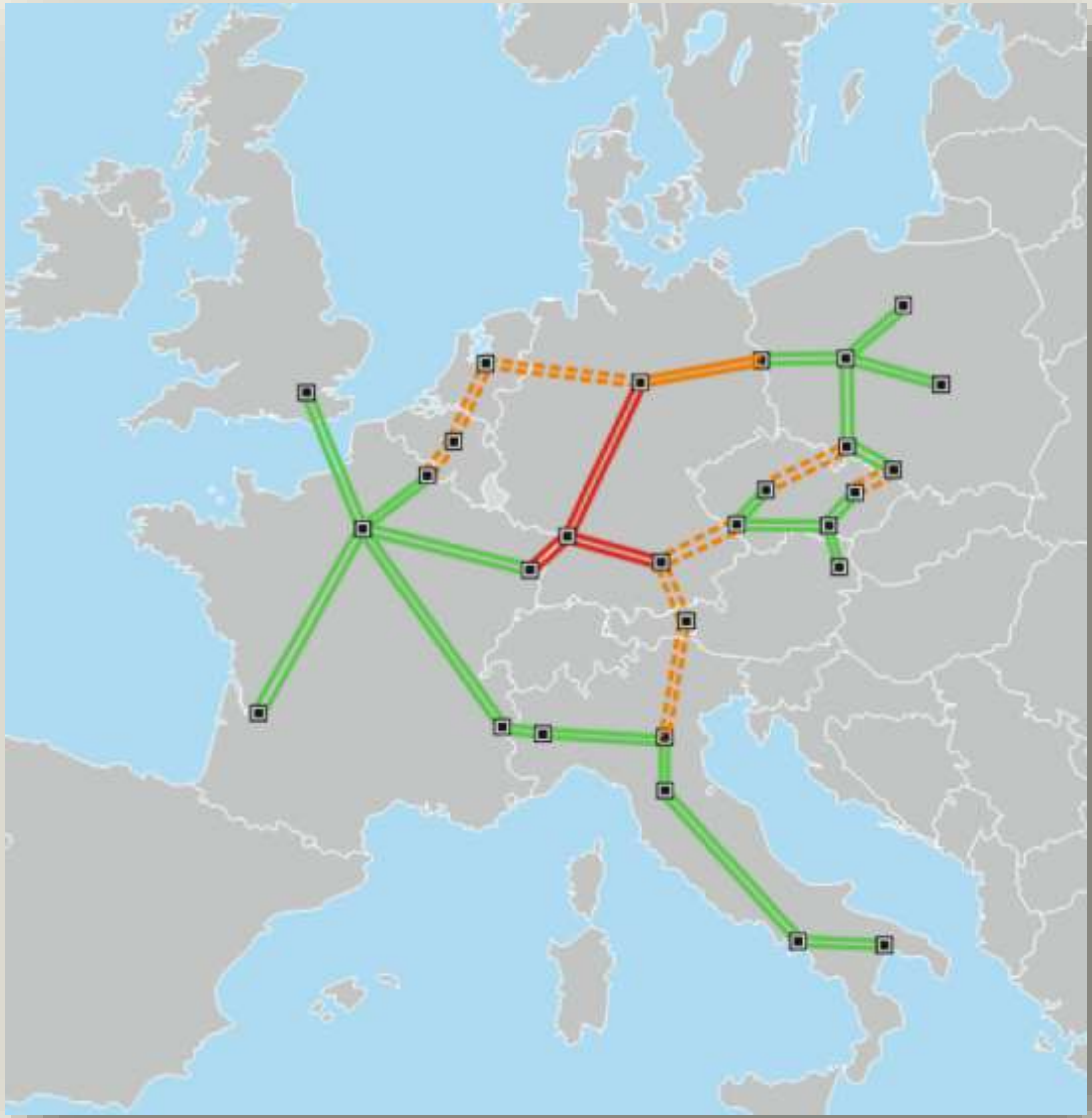
Optical frequency dissemination

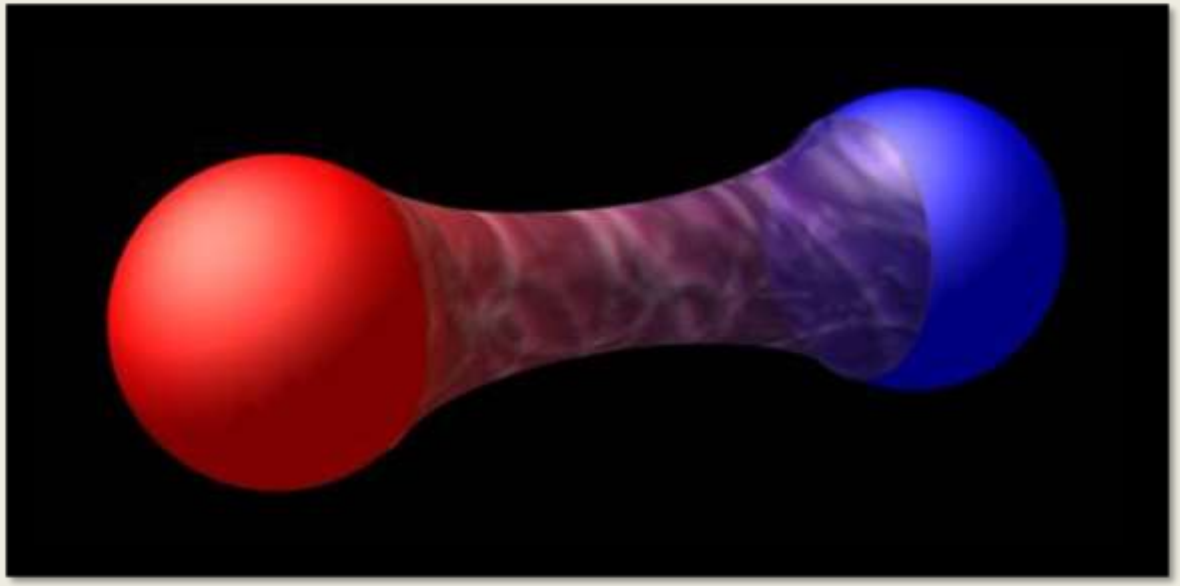
- Fiber links connect clocks in Europe
- Intercontinental links are an open issue.

→ Optical free-space instead of microwave satellite links?



Free-space links





QUANTUM LOGIC SPECTROSCOPY: MOTIVATION

Wishlist for Ideal Clock Atom

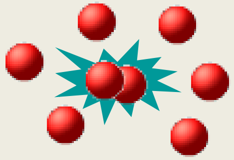
- **High stability**

  quantum projection noise limit: $\sigma_y(\tau) = \frac{1}{2\pi f_0 \sqrt{N T_R \tau}}$

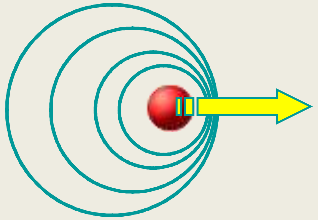
  ➔ narrow optical transition, many atoms

- **High accuracy**

  ➔ low sensitivity to resonance shifts

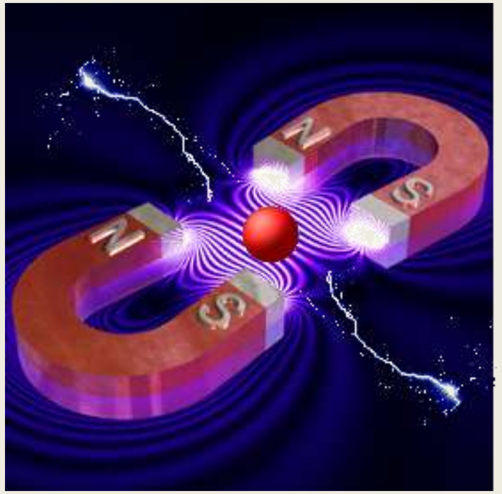


collisions



Doppler
shifts

electric and
magnetic fields

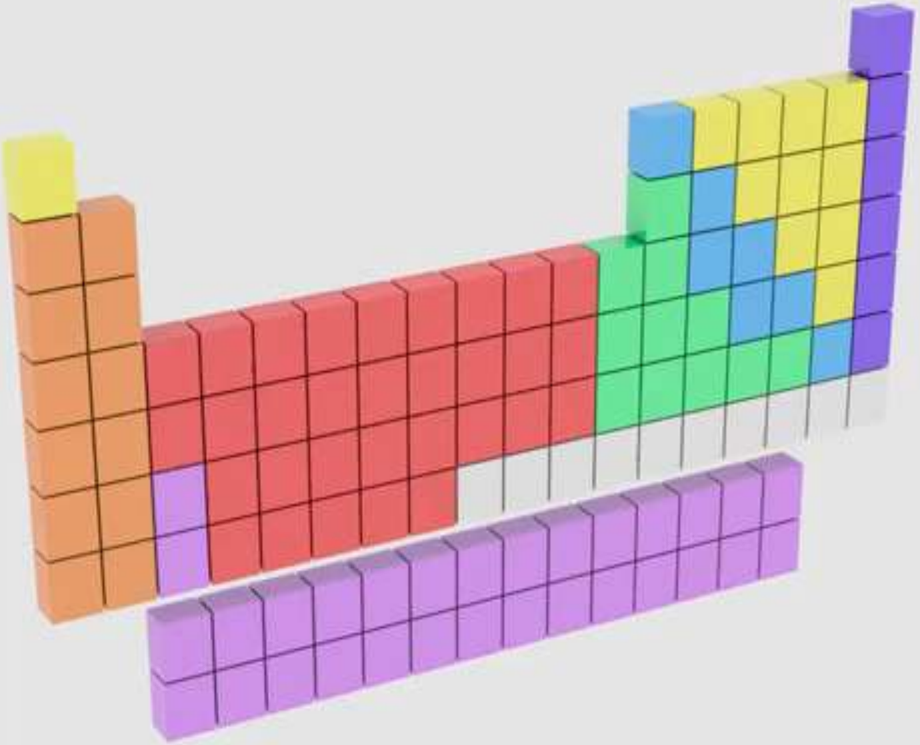


- **Many species to choose from:**

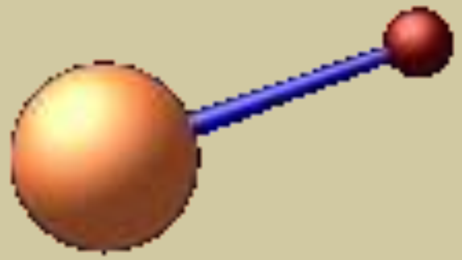
  Neutral atoms: Ca, Mg, Sr, Yb, Hg, Ag

  Ions: Hg^+, Sr^+, Yb^+, In^+, Ca^+, Al^+

Available species?

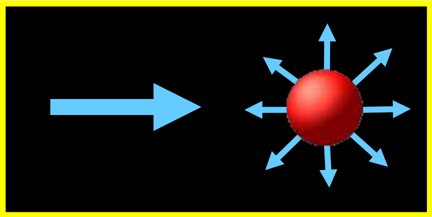


Molecules?

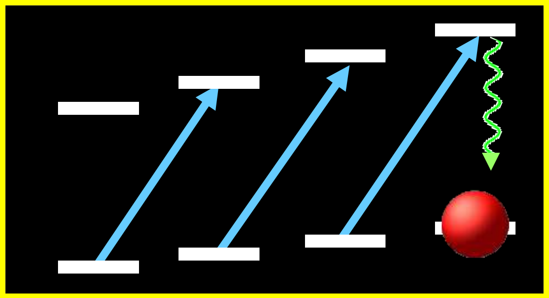


A typical (spectroscopy) sequence

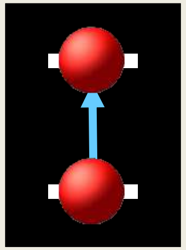
- minimize motion
  → laser cooling



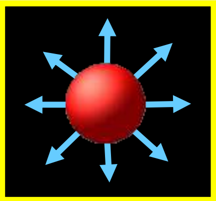
- internal state preparation
  →optical pumping

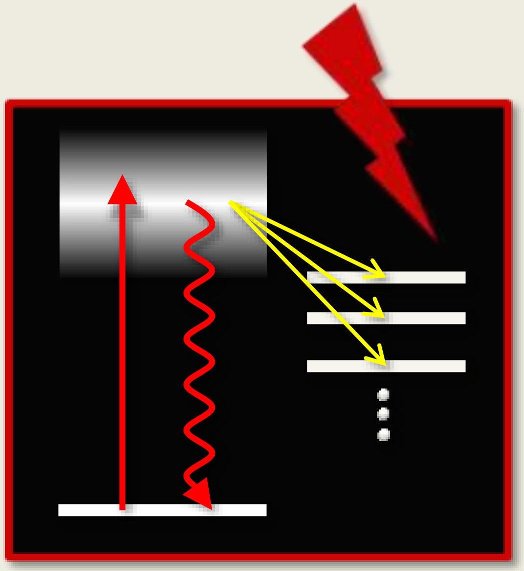


- interrogation
  → probe spectroscopy transition



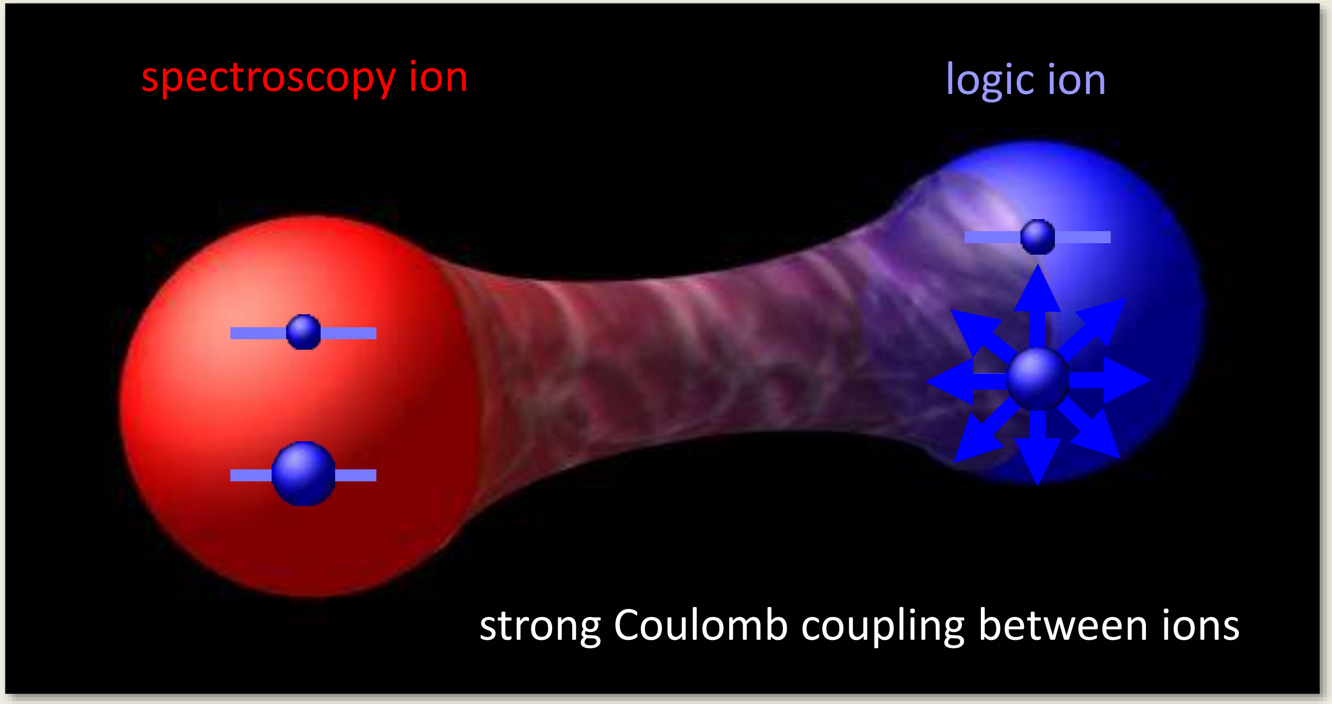
- internal state detection
  → state dependent fluorescence





→ Need laser cooling transition!

Quantum Logic Spectroscopy

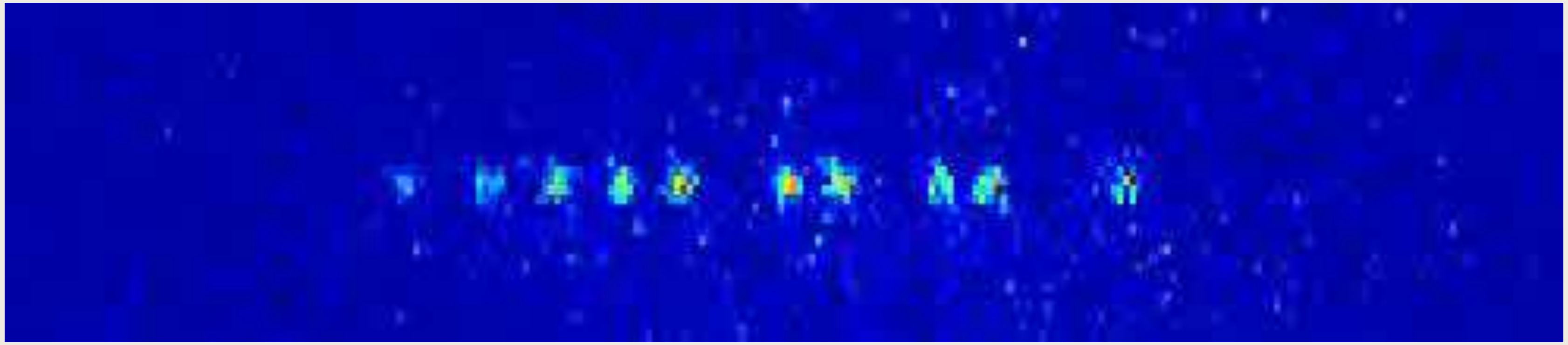


- ions in linear Paul trap → high accuracy achievable
- logic ion provides sympathetic cooling & signal readout
- strong Coulomb interaction couples motional modes
- composite system: combine advantages of both species

→ investigation of previously inaccessible species

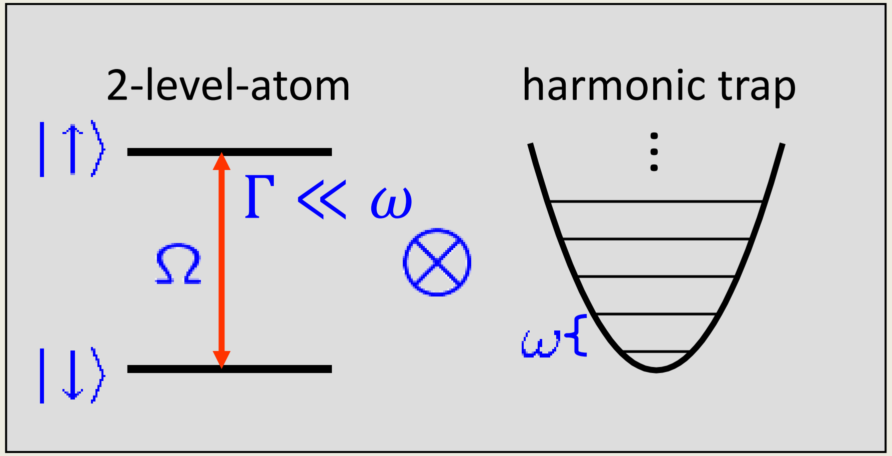
[D.J. Wineland *et. al.*, Proc. 6th Symposium on Frequency Standards and Metrology, 361 (2001); P.O. Schmidt *et al.*, *Science*, **309**, 749 (2005)]

Sympathetic Cooling



string of Mg^+ and other ions (MgH^+) in linear Paul trap

➔ a (mostly) solved problem



QUANTUM LOGIC WITH TRAPPED IONS

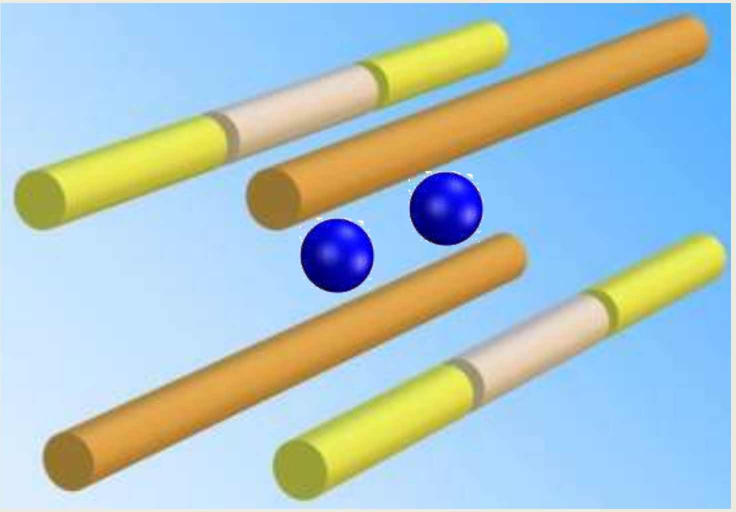
Quantum Logic with Trapped Ions

- Idea by:

J. I. Cirac

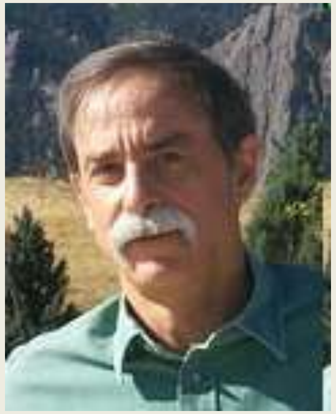
P. Zoller

PRL **74**, 4091 (1995)



Collective motion of ions described by normal modes

D. Wineland



NP 2012

- implementation of QC
- idea of quantum logic spectroscopy

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

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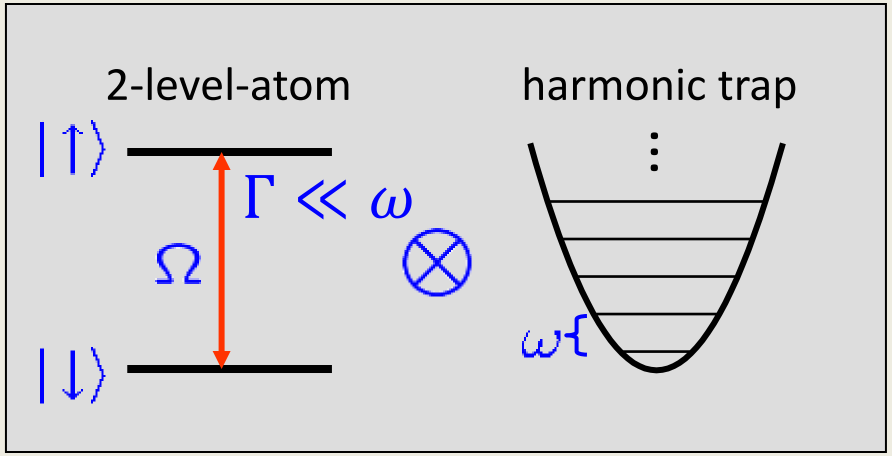
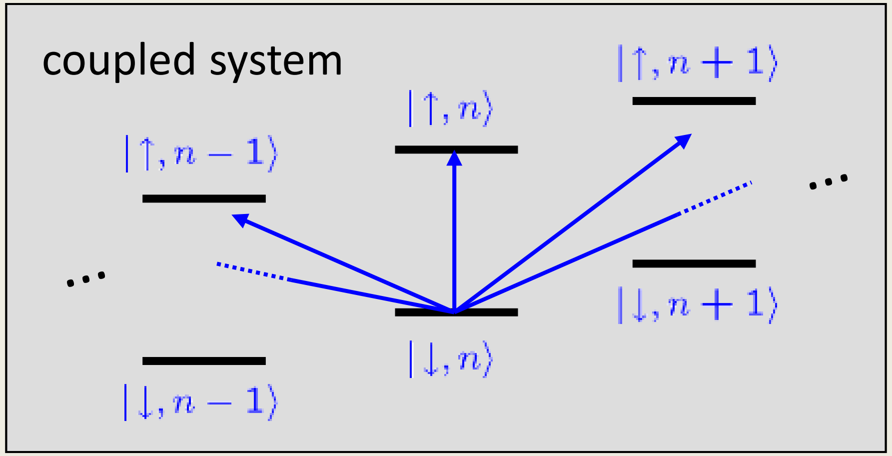
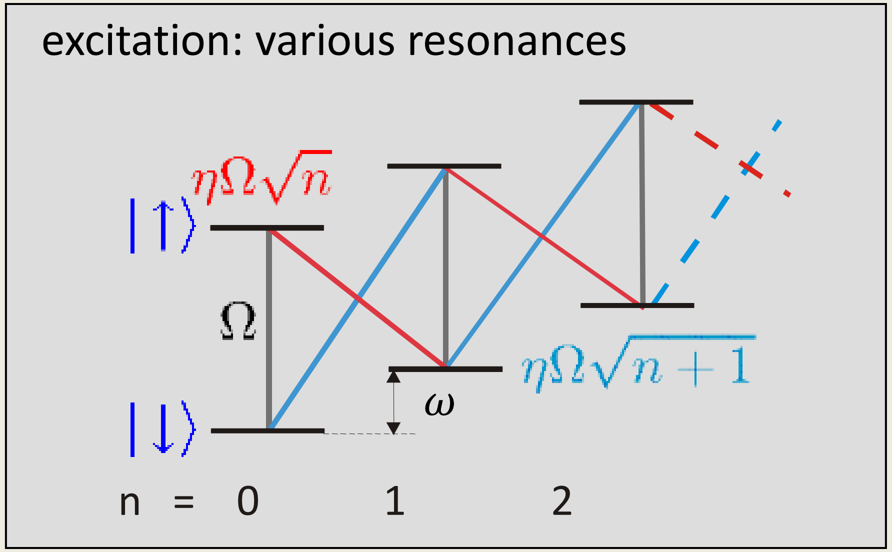
(Received 30 November 1994)

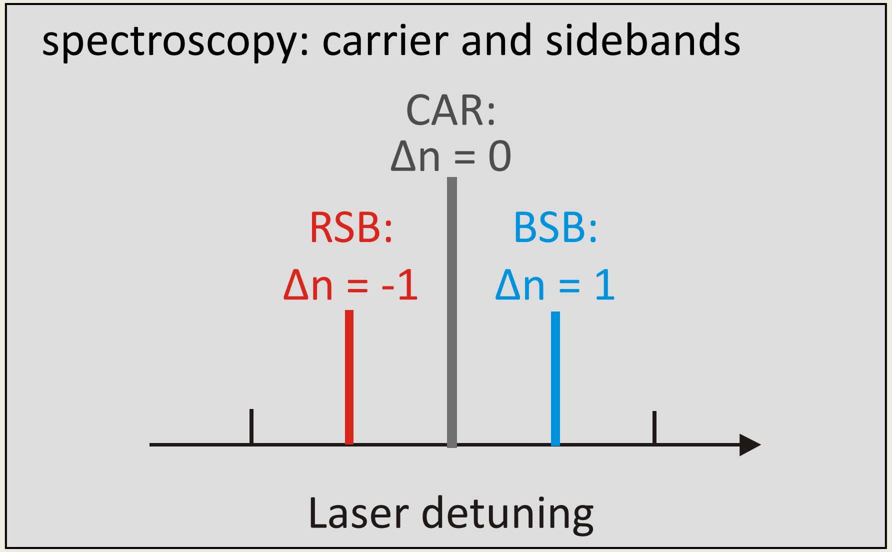
A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

Quantum Logic with Trapped Ions



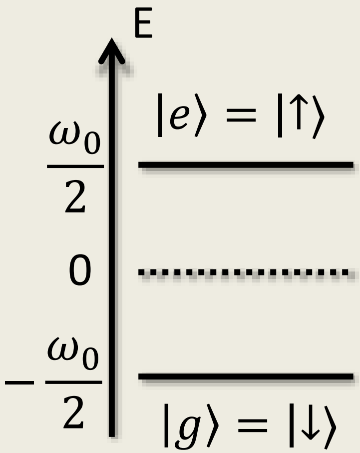






Ω: Carrier Rabi frequency; $\eta = kz_0$: Lamb-Dicke factor

Atom-light interaction



- Hamiltonian: $H = H_a + H_m + H_i$
- Atom: $H_a = \frac{\hbar\omega_o}{2}(|e\rangle\langle e| - |g\rangle\langle g|) = \frac{\hbar\omega_0}{2}\sigma_z$
- Motion: $H_m = \hbar\omega_z a^\dagger a$
- Atom-Light-Interaction: $H_i = -\hat{\vec{d}}\vec{E} = e\hat{\vec{r}}\vec{E}_0\cos(k\hat{z} - \omega t + \phi)$

$$\Rightarrow \mathrm{H_i} = \frac{\hbar\Omega}{2}(\sigma_+ + \sigma_-)\left(e^{i(k\hat{z}-\omega t+\phi)} + e^{-i(k\hat{z}-\omega t+\phi)}\right)$$

with $\Omega = \Omega_{ge} = \frac{e\vec{E}_0}{\hbar}\langle e|\hat{\vec{r}}|g\rangle$ and $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, Basis: $\{|g,n\rangle, |e,n\rangle\}$

→ quantum dynamics simulations using QuTiP

[D. J. Wineland *et al.*, J. Res. Natl. Inst. Stand. Technol. **103**, 259–328 (1998)]

Coupling between motional states

- similar to Franck-Condon in molecular transitions:

Laguerre polynnomials

$$\langle n_j | e^{ik\hat{z}} | n_l \rangle = \langle n_j \left| e^{i\eta(\hat{a}+\hat{a}^\dagger)} \right| n_l \rangle = e^{-\frac{\eta^2}{2}} \sqrt{\frac{n_<!}{n_>!}} \eta^{\Delta n} L_{n_<}^{\Delta n}(\eta^2) \qquad \Delta n = |n_j - n_i|$$

$n_<$ ($n_>$): lesser (greater) of $n_{j,l}$

using $k\hat{z} = \eta(\hat{a} + \hat{a}^\dagger)$ (more general: $\vec{k}\hat{\vec{z}}$)

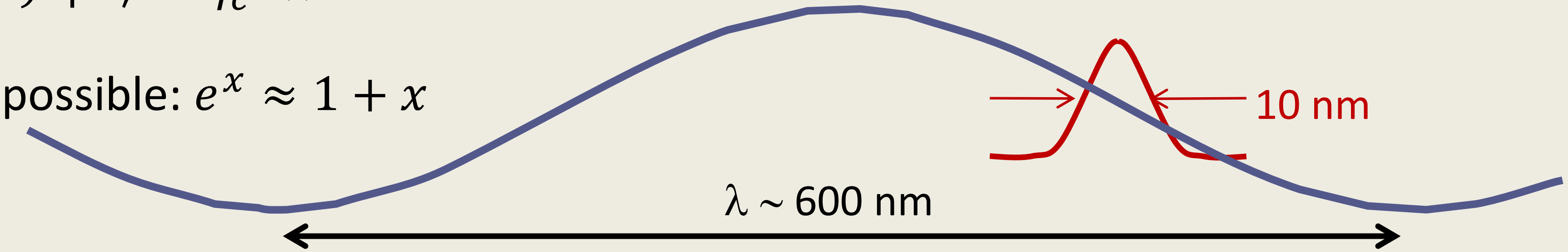
→ Lamb-Dicke parameter: $\eta = kz_0 = k\sqrt{\frac{\hbar}{2m\omega_z}}$

- typical values for Lamb-Dicke parameter $\eta = kz_0$: $z_0 \sim 10$ nm → $\eta \sim 0.1$
- simplification in Lamb-Dicke regime:

Motional wavefunction

$$(k\langle \hat{z}^2 \rangle)^{1/2} = \eta\sqrt{\langle\Psi|(\hat{a} + \hat{a}^\dagger)^2|\Psi\rangle} = \eta_c \ll 1$$

→ expansion of exponential possible: $e^x \approx 1 + x$



[D. J. Wineland *et al.*, J. Res. Natl. Inst. Stand. Technol. **103**, 259–328 (1998)]

Quantized atom-light interaction including motion

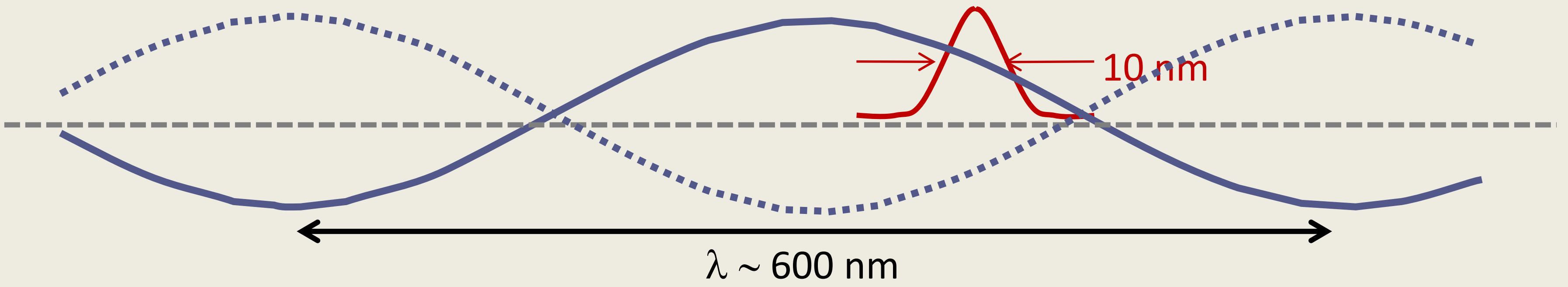
- Interaction in Lamb-Dicke regime:

$$H_i' \approx \left(\hbar\Omega\hat{\sigma}_+ e^{-i\Delta t+i\phi}\right)/2 + h.c.$$ carrier (Δn=0)

$$+\left(i\hbar\eta\Omega\hat{\sigma}_+\hat{a}e^{-i(\Delta+\omega_z)t+i\phi}\right)/2 + h.c.$$ red sideband

$$+\left(i\hbar\eta\Omega\hat{\sigma}_+\hat{a}^\dagger e^{-i(\Delta-\omega_z)t+i\phi}\right)/2 + h.c.$$ blue sideband

with $\eta = kz_0$ and $z_0 = \sqrt{\hbar/2m\omega_z}$



- $E = const$ drives carrier
- $\vec{\nabla}E$ drives sidebands (smaller by η)

What happens for
$(k\langle\hat{z}^2\rangle)^{1/2} = \eta\sqrt{\langle\Psi|(\hat{a}+\hat{a}^\dagger)^2|\Psi\rangle} = \eta_c > 1$?

[D. J. Wineland *et al.*, J. Res. Natl. Inst. Stand. Technol. **103**, 259–328 (1998)]

Summary Quantum Logic Spectroscopy & Clocks I

- Optical clocks based have reached systematic uncertainties of 10^{-18}
- Ion clocks have low systematic uncertainty
- Neutral atom lattice clocks have a high stability
- Different optical clocks can be compared using an optical frequency comb
- Optical clocks at different locations can be compared using length-stabilized fiber links
- Clock spectroscopy requires cycling transition for cooling & detection
  → Quantum logic techniques from quantum computing:
  Quantum Logic Spectroscopy