Lectures #6 - 7

PARITY NONCONSERVATION IN ATOMS

Review paper:

G.S.M. Ginges and V.M. Flambaum
"*Violations of fundamental symmetries in atoms and tests of unification theories of elementary particles*"
arXiv: physics/0309054 (submitted to the Physics Reports)

Research topic example: Parity nonconservation

Topics covered

Conservation laws

Perturbation theory

Angular momentum addition

Clebsch-Gordon coefficients
3-j symbols
6-j symbols

Angular momentum diagrams

Wigner-Eckart theorem

Electric-dipole transitions

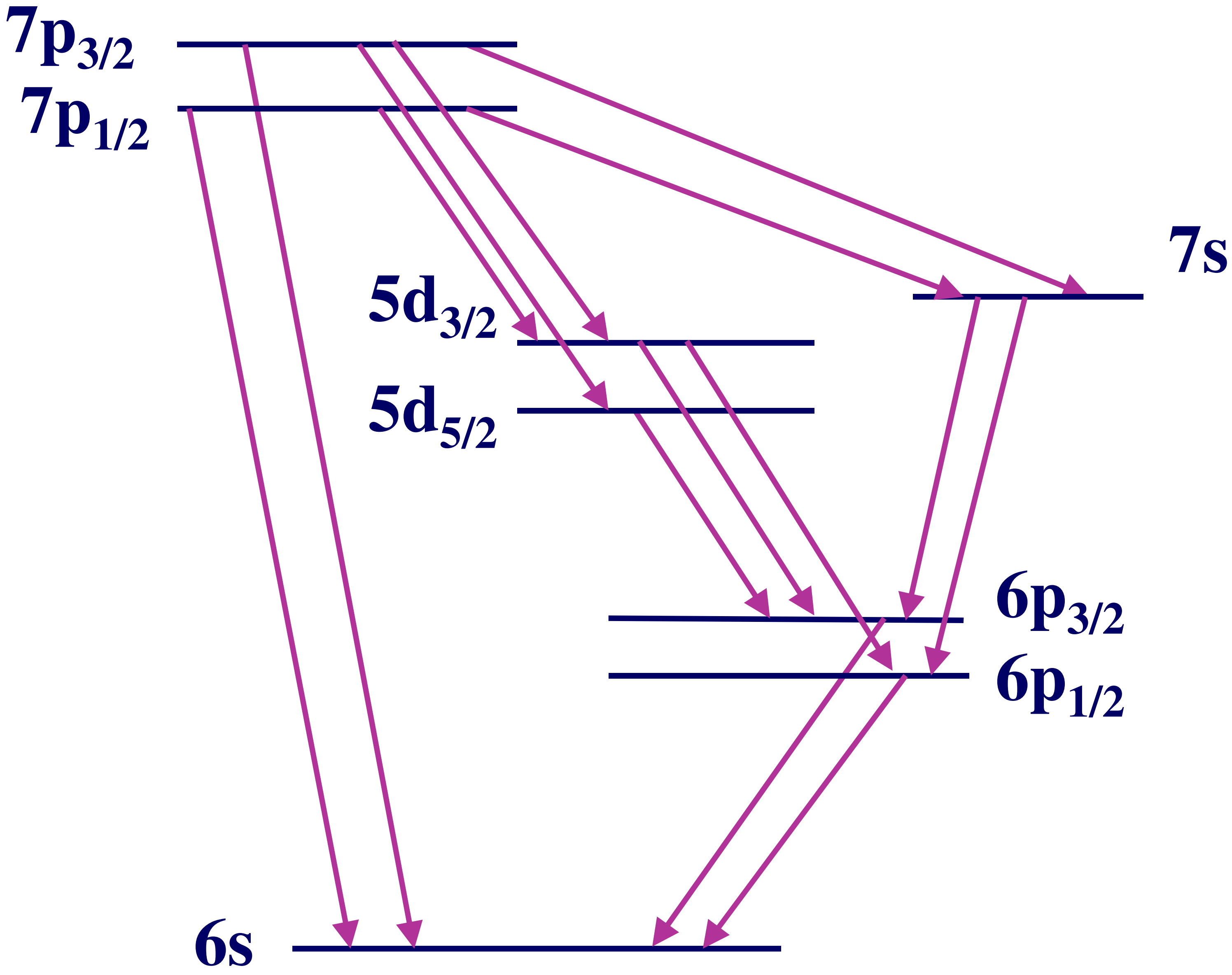
Advanced methods of atomic calcualations

Irreducible tensors and their matrix elements

Hyperfine structure

Weak interactions

Cs electric-dipole transitions

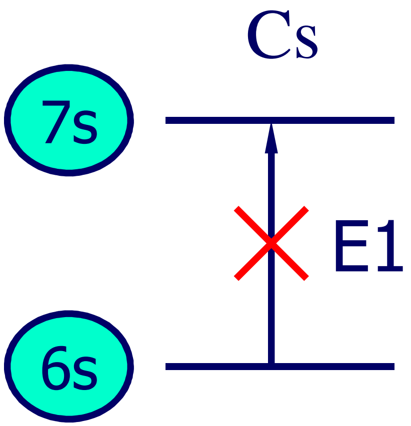


Sources of parity violation in atoms

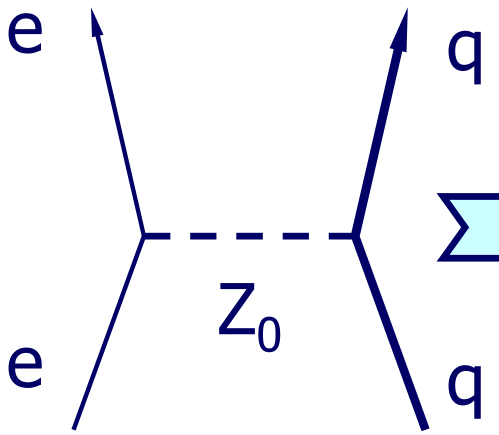
The main source of parity-nonconserving (PNC) effects in atoms is the exchange of a virtual Z_0 boson between atomic electrons and the nucleus.

There are several others, the main nuclear-spin dependent effect results from the nuclear anapole (parity-violating) moment.

Parity nonconservation (PNC)



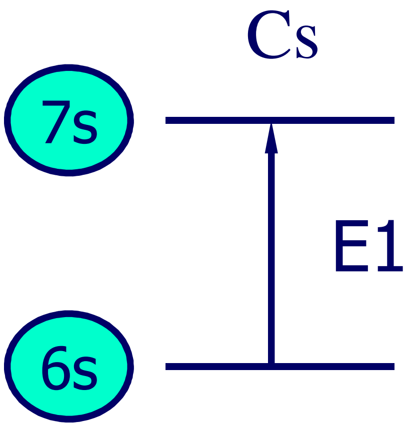
Forbidden by parity selection rules



Z_0 exchange: Laporte’s rule is violated!

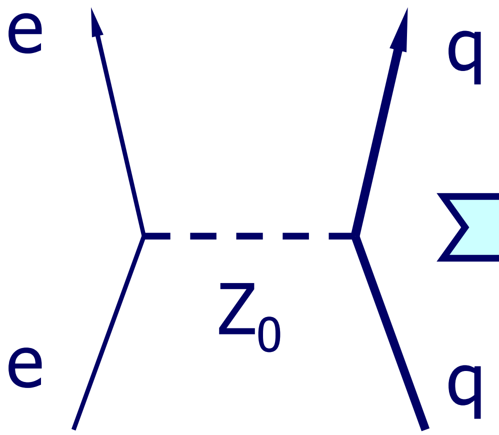
Laporte’s rule: parity selection rule

Parity nonconservation (PNC)



Non-zero transition amplitude

Both 6s and 7s states acquire an opposite-parity ($np_{1/2}$) admixture



Z_0 exchange: Laporte’s rule is violated!

Laporte’s rule: parity selection rule

How to construct a PNC Hamiltonian?

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$H^{(1)} = A_e V_N$ product of axial-vector electron A_e and vector nucleon $\mathrm{V_N}$ currents

$$H^{(1)} = \frac{G}{\sqrt{2}} \left(\bar{\psi}_e \gamma_\mu \gamma_5 \psi_e \right) \sum_i \left[c_{1p} \left(\bar{\psi}_{pi} \gamma^\mu \psi_{pi} \right) + c_{1n} \left(\bar{\psi}_{ni} \gamma^\mu \psi_{ni} \right) \right]$$

Coupling constants

$$H^{(2)} = \frac{G}{\sqrt{2}} \left(\bar{\psi}_e \gamma_\mu \psi_e \right) \sum_i \left[c_{2p} \left(\bar{\psi}_{pi} \gamma^\mu \gamma_5 \psi_{pi} \right) + c_{1n} \left(\bar{\psi}_{ni} \gamma^\mu \gamma_5 \psi_{ni} \right) \right]$$

$H^{(2)} = V_e A_N$ product of vector electron V_e and axial-vector nucleon A_N currents

How to construct a PNC Hamiltonian?

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Assume that nucleons are nonrelativistic →

$$\left(\bar{\psi}_{pi}\gamma^{\mu}\psi_{pi}\right) \to \phi_p^{\dagger}\phi_p\delta_{\mu 0}$$

$$\left(\bar{\psi}_{ni}\gamma^{\mu}\psi_{ni}\right) \to \phi_n^{\dagger}\phi_n\delta_{\mu 0}$$

Time-like (μ=0) component of $H_{(1)}$ yields for the "effective" Hamiltonian in the electron sector:

$$\int d^3r\,\rho_{n,p}(r) = 1$$

$$H_{eff}^{(1)} = \frac{G}{\sqrt{2}}\gamma_5\left[c_{1p}Z\rho_p(r) + c_{1n}N\rho_n(r)\right]$$

number of protons

number of neutrons

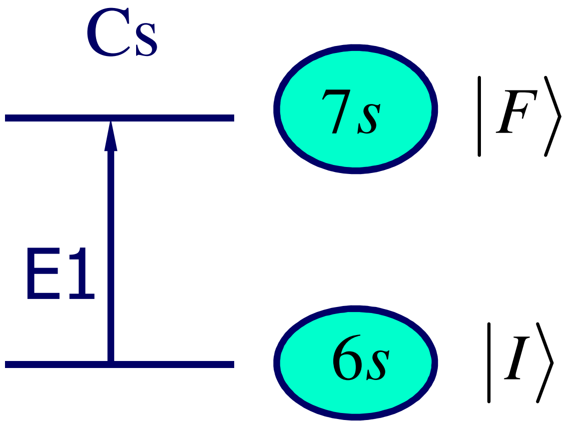
Assume $\rho_n(r) = \rho_p(r) \equiv \rho(r)$ $\Rightarrow$

$$H_{eff}^{(1)} = \frac{G}{2\sqrt{2}}\gamma_5\underbrace{\left[2c_{1p}Z + 2c_{1n}N\right]}_{Q_W}\rho(r)$$

$$\boxed{H_{eff}^{(1)} = \frac{G}{2\sqrt{2}}\gamma_5 Q_W\,\rho(r)}$$

← Weak charge → $\boxed{Q_W}$

Parity nonconservation



Lets only consider only this PNC effect for now and call the corresponding Hamiltonian H_{PNC}

$$H_{PNC} = H^{(1)}_{eff} = \frac{G}{2\sqrt{2}}\gamma_5 Q_W\ \rho(r)$$

It is the dominant PNC effect but there are several others [described by $H^{(2)}$, for example].

How do we calculate the corresponding dipole matrix element?

(How do we get p state admixtures to 6s and 7s states?)

$$\langle F|d|I\rangle = ?$$

Use perturbation theory

Time-independent perturbation theory

Suppose this is the problem we know how to solve:

$$H\left|\psi_n\right\rangle = E_n\left|\psi_n\right\rangle \qquad \left\langle\psi_n\middle|\psi_m\right\rangle = \delta_{mn}$$

We found complete set of eigenfunctions and eigenvalues.

Let it be our starting point.

rename to $^{(0)}$

$$H^{(0)}\left|\psi_n^{(0)}\right\rangle = E_n^{(0)}\left|\psi_n^{(0)}\right\rangle \qquad \left\langle\psi_n^{(0)}\middle|\psi_m^{(0)}\right\rangle = \delta_{mn}$$

The problem which we want to solve is

$$\boxed{\begin{gathered} H\left|\psi_n\right\rangle = E_n\left|\psi_n\right\rangle \\ H = H^{(0)} + H\,' \end{gathered}}$$

Small perturbation

Time-independent perturbation theory

$$H\left|\psi_n\right\rangle = E_n\left|\psi_n\right\rangle$$

$$H = H^{(0)} + H'$$

Lets expand our unknown wave functions and energies as

Main idea of the perturbation theory

$$\left|\psi_n\right\rangle = \left|\psi_n^{(0)}\right\rangle + \left|\psi_n^{(1)}\right\rangle + \left|\psi_n^{(2)}\right\rangle + \ldots$$

$$E_n = E_n^{(0)} + E_n^{(1)} + E_n^{(2)} + \ldots$$

Already known solutions of

$$H^{(0)}\left|\psi_n^{(0)}\right\rangle = E_n^{(0)}\left|\psi_n^{(0)}\right\rangle$$

Some results of the perturbation theory

How to calculate first order energy correction:

$$E_n^{(1)} = \left\langle \psi_n^{(0)} \middle| H' \middle| \psi_n^{(0)} \right\rangle$$

First order wave functions are given by:

$$\left| \psi_n^{(1)} \right\rangle = \sum_{m \neq n} \frac{\left| \psi_m^{(0)} \right\rangle \left\langle \psi_m^{(0)} \middle| H' \middle| \psi_n^{(0)} \right\rangle}{E_n^{(0)} - E_n^{(0)}}$$

We now apply this formula to our parity nonconservation example

PNC & perturbation theory

$$H^{(0)}\left|\psi_n^{(0)}\right\rangle = E_n^{(0)}\left|\psi_n^{(0)}\right\rangle \qquad \left\langle\psi_n^{(0)}\middle|\psi_m^{(0)}\right\rangle = \delta_{mn}$$

Our known (relativistic) wave functions
(may be calculated using some very high-precision method
such as all-order method or may be taken as some
basic approximation, such as Dirac-Hartree-Fock)
For the application of the perturbation theory it does not matter
how well did we solve the initial problem,
we just need to have some solution

$$H' = H_{PNC}$$

$$\left|\psi_n\right\rangle = \left|\psi_n^{(0)}\right\rangle + \left|\psi_n^{(1)}\right\rangle \qquad \left|\psi_n^{(1)}\right\rangle = \sum_{m\neq n}\frac{\left|\psi_m^{(0)}\right\rangle\left\langle\psi_m^{(0)}\right|H_{PNC}\left|\psi_n^{(0)}\right\rangle}{E_n^{(0)} - E_m^{(0)}}$$

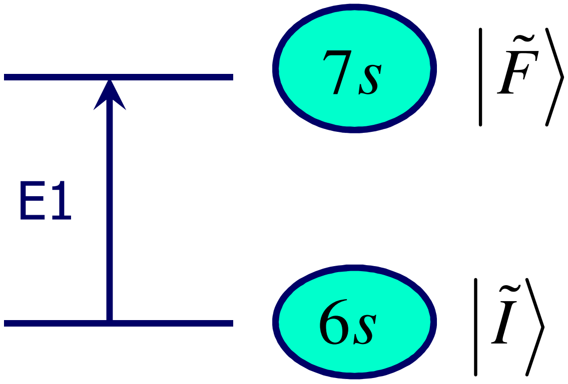
A bit simpler designations

$$|\psi_n\rangle = |\psi_n^{(0)}\rangle + |\psi_n^{(1)}\rangle \qquad |\psi_n^{(1)}\rangle = \sum_{m\neq n} \frac{|\psi_m^{(0)}\rangle\langle\psi_m^{(0)}|H_{PNC}|\psi_n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}}$$

$$|\tilde{n}\rangle = |n\rangle + \sum_{m\neq n} \frac{|m\rangle\langle m|H_{PNC}|n\rangle}{\varepsilon_n - \varepsilon_m}$$

Perturbed wave functions (to first order)

Initial wave functions

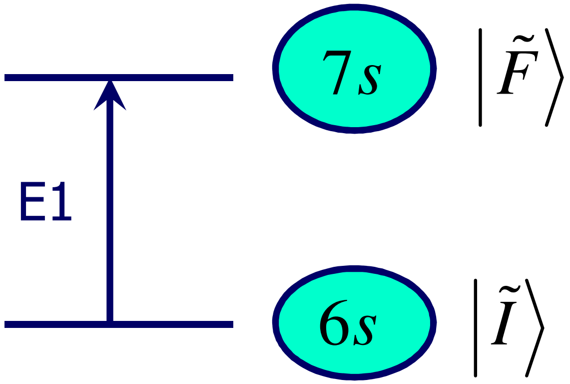


$$\underbrace{\langle F|d|I\rangle}_{\text{Initial wave functions, no perturbation}} = 0 \xrightarrow{PNC} \boxed{\langle\tilde{F}|d|\tilde{I}\rangle = \langle\tilde{F}|d|I\rangle + \langle F|d|\tilde{I}\rangle}$$

7s-6s electric-dipole (E1) matrix element in Cs

$$\left\langle \tilde{F} \middle| d \middle| \tilde{I} \right\rangle = \left\langle \tilde{F} \middle| d \middle| I \right\rangle + \left\langle F \middle| d \middle| \tilde{I} \right\rangle$$

$$\left| \tilde{n} \right\rangle = \left| n \right\rangle + \sum_{m \neq n} \frac{\left| m \right\rangle \left\langle m \middle| H_{PNC} \middle| n \right\rangle}{\varepsilon_n - \varepsilon_m}$$

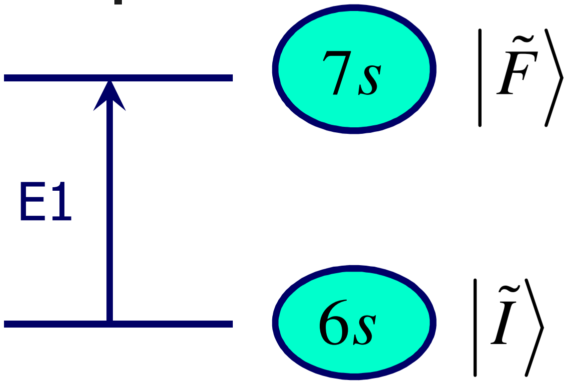


$$\left\langle \tilde{F} \middle| d \middle| \tilde{I} \right\rangle = \left\langle \tilde{F} \middle| d \middle| I \right\rangle + \left\langle F \middle| d \middle| \tilde{I} \right\rangle$$

$$\left\langle \tilde{F} \middle| d \middle| \tilde{I} \right\rangle = \overbrace{\sum_{N \neq F} \frac{\left\langle F \middle| H_{PNC} \middle| N \right\rangle \left\langle N \middle| d \middle| I \right\rangle}{\varepsilon_F - \varepsilon_N}} + \overbrace{\sum_{N \neq I} \frac{\left\langle F \middle| d \middle| N \right\rangle \left\langle N \middle| H_{PNC} \middle| I \right\rangle}{\varepsilon_I - \varepsilon_N}}$$

Cs

Quantum numbers?



$$\langle \tilde{F} | d | \tilde{I} \rangle = \sum_{N \neq F} \frac{\langle F | H_{PNC} | N \rangle \langle N | d | I \rangle}{\varepsilon_F - \varepsilon_N} + \sum_{N \neq I} \frac{\langle F | d | N \rangle \langle N | H_{PNC} | I \rangle}{\varepsilon_I - \varepsilon_N}$$

What quantum numbers describe $|F\rangle$, $|I\rangle$, and $|N\rangle$?

Our PNC Hamiltonian is nuclear-spin independent so we can take only $|n\, l\; j\, m_j\rangle$

l is included to indicate parity of the state.

$$|F\rangle = |n=7\; l=0\; j=1/2\; m_F\rangle = |7\text{s}\; m_F\rangle$$

$$|I\rangle = |n=6\; l=0\; j=1/2\; m_I\rangle = |6\text{s}\; m_I\rangle$$

$$|N\rangle = |n\; l\; j\; m_N\rangle$$

Note s state has only j=1/2 so we write

$$6s \equiv 6s_{1/2}$$

$$7s \equiv 7s_{1/2}$$

Which intermediate states N are possible?

Cs

$$\left\langle \widetilde{7sm_F} \middle| d \middle| \widetilde{6sm_I} \right\rangle = \sum_{N \neq F} \frac{\left\langle 7s\, m_F \middle| H_{PNC} \middle| nljm_n \right\rangle \left\langle nljm_n \middle| d \middle| 6s\, m_I \right\rangle}{\varepsilon_{7s} - \varepsilon_N}$$

$$+ \sum_{N \neq I} \frac{\left\langle 7s\, m_F \middle| d \middle| nljm_n \right\rangle \left\langle nljm_n \middle| H_{PNC} \middle| 6s\, m_I \right\rangle}{\varepsilon_{6s} - \varepsilon_N}$$

Lets look at our H_{PNC}: $$H_{PNC} = \frac{G}{2\sqrt{2}} \gamma_5 Q_W\, \rho(r)$$

It is : a) a scalar operator $\Rightarrow \left\langle j_a m_a \middle| H_{PNC} \middle| j_b m_b \right\rangle = 0 \;\; if \;\; j_a \neq j_b, m_a \neq m_b$

b) changes parity

Therefore, N must be $np_{1/2}$ state since 7s and 6s have j=1/2

Reduced matrix elements and sum over m_n

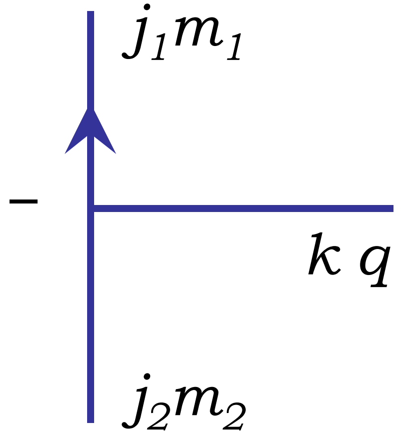
Cs

$$\left\langle \widetilde{7sm_F} \middle| d \middle| \widetilde{6sm_I} \right\rangle = \sum_{nm_n} \frac{\langle 7s\, m_F | H_{PNC} | np_{1/2} m_n \rangle \langle np_{1/2} m_n | d | 6s\, m_I \rangle}{\varepsilon_{7s} - \varepsilon_{np1/2}}$$

$$+ \sum_{nm_n} \frac{\langle 7s\, m_F | d | np_{1/2} m_n \rangle \langle np_{1/2} m_n | H_{PNC} | 6s\, m_I \rangle}{\varepsilon_{6s} - \varepsilon_{np1/2}}$$

We use Wigner-Eckart theorem to sum over m_n

$$\langle j_1 m_1 | T^k_q | j_2 m_2 \rangle =$$



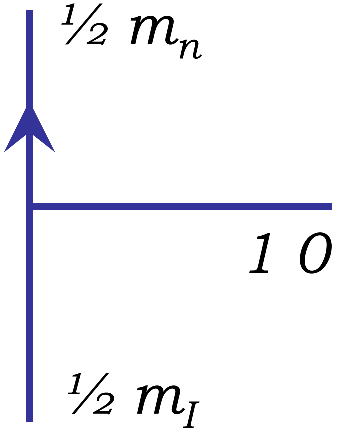
$$\langle j_1 \| T^k \| j_2 \rangle$$

Reduced matrix element
Does not depend on m's

Dipole matrix elements

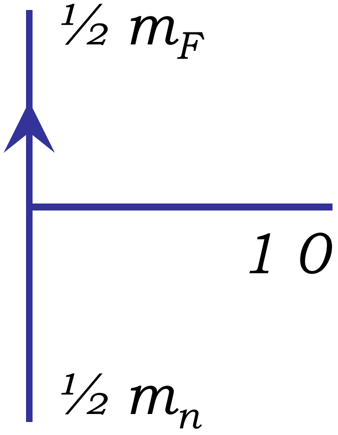
d=ez, k=1, q=0

$$\langle np_{1/2} m_n \left| d \right| 6s\, m_I \rangle = -$$



$$\langle np_{1/2} \| d \| 6s \rangle$$

$$\langle 7s\, m_F \left| d \right| np_{1/2} m_n \rangle = -$$



$$\langle 7s \| d \| np_{1/2} \rangle$$

PNC matrix elements

k=0, q=0

$$H_{PNC} = \frac{G}{2\sqrt{2}} \gamma_5 Q_W \; \rho(r) : \qquad scalar \;\; k = 0, \;\; q = 0$$

Warning: this PNC matrix element has non-standard definition for the reduced matrix elements

$$\left\langle j_a m_a \left| H_{PNC} \right| j_b m_b \right\rangle = \delta_{j_a j_b} \, \delta_{m_a m_b} \; \left\langle j_a \left\| H_{PNC} \right\| j_b \right\rangle$$

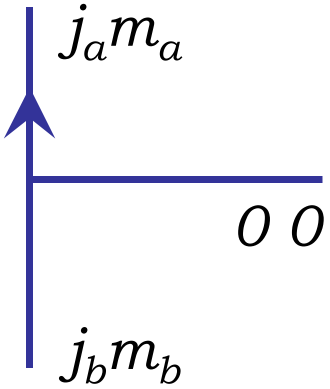
Why is this "non-standard"?

$$H_{PNC} = \frac{G}{2\sqrt{2}}\gamma_5 Q_W \,\rho(r): \qquad scalar\ \ k=0,\ \ q=0$$

Non-standard definition →

$$\boxed{\left\langle j_a m_a \left| H_{PNC} \right| j_b m_b \right\rangle = \delta_{j_a j_b}\,\delta_{m_a m_b}\,\left\langle j_a \left\| H_{PNC} \right\| j_b \right\rangle}$$

$$\left\langle j_a m_a \left| H_{PNC} \right| j_b m_b \right\rangle = -$$



$$\left\langle j_a \left\| H_{PNC} \right\| j_b \right\rangle$$

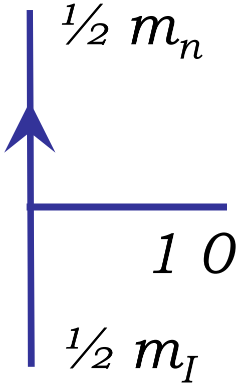
Extra factor

$$(-1)^{j_a - m_a}\begin{pmatrix} j_a & 0 & j_b \\ -m_a & 0 & m_b \end{pmatrix} = \delta_{j_a j_b}\,\delta_{m_a m_b}\,\frac{1}{\sqrt{2j_a+1}}$$

Putting it all together: Term 1

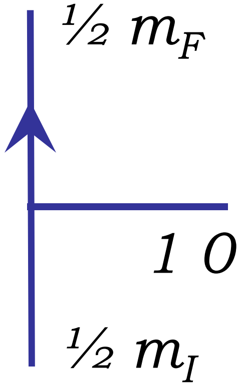
$$\sum_{nm_n}\frac{\langle 7s\, m_F\left|H_{PNC}\right|np_{1/2}m_n\rangle\langle np_{1/2}m_n\left|d\right|6s\, m_I\rangle}{\varepsilon_{7s}-\varepsilon_{np_{1/2}}}$$

$$=\sum_{nm_n} -$$



$$\delta_{m_n m_F}\frac{\langle 7s\left\|H_{PNC}\right\|np_{1/2}\rangle\langle np_{1/2}\left\|d\right\|6s\rangle}{\varepsilon_{7s}-\varepsilon_{np_{1/2}}}$$

$$= \quad -$$

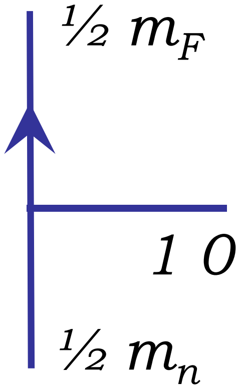


$$\sum_{n}\frac{\langle 7s\left\|H_{PNC}\right\|np_{1/2}\rangle\langle np_{1/2}\left\|d\right\|6s\rangle}{\varepsilon_{7s}-\varepsilon_{np_{1/2}}}$$

Putting it all together: Term 2

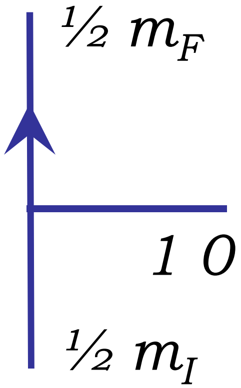
$$\sum_{nm_n} \frac{\langle 7s\, m_F \left| d \right| np_{1/2} m_n \rangle \langle np_{1/2} m_n \left| H_{PNC} \right| 6s\, m_I \rangle}{\varepsilon_{7s} - \varepsilon_{np_{1/2}}}$$

$$= \sum_{nm_n} -$$



$$\delta_{m_n m_I} \frac{\langle 7s \| d \| np_{1/2} \rangle \langle np_{1/2} \| H_{PNC} \| 6s \rangle}{\varepsilon_{7s} - \varepsilon_{np_{1/2}}}$$

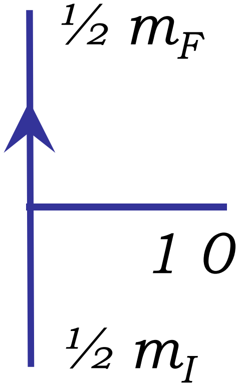
$$= \quad -$$



$$\sum_{n} \frac{\langle 7s \| d \| np_{1/2} \rangle \langle np_{1/2} \| H_{PNC} \| 6s \rangle}{\varepsilon_{7s} - \varepsilon_{np_{1/2}}}$$

Putting it all together

$$\left\langle \widetilde{7sm_F}\,\middle|\,d\,\middle|\,\widetilde{6sm_I}\right\rangle = \quad -$$



$$\sum_{\mathrm{n}} \left\{ \frac{\left\langle 7s \left\| d \right\| np_{1/2} \right\rangle \left\langle np_{1/2} \left\| H_{PNC} \right\| 6s \right\rangle}{\varepsilon_{7s} - \varepsilon_{np1/2}} + \frac{\left\langle 7s \left\| H_{PNC} \right\| np_{1/2} \right\rangle \left\langle np_{1/2} \left\| d \right\| 6s \right\rangle}{\varepsilon_{7s} - \varepsilon_{np1/2}} \right\}$$

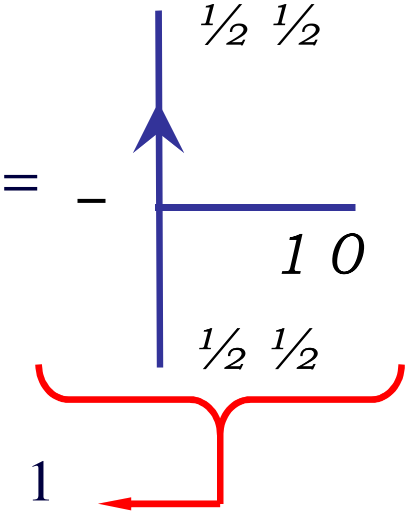
Definition of the PNC amplitude E_{PNC}: maximum m_F and m_I

$$E_{PNC} = \left\langle \widetilde{7sm_F}\,\middle|\,d\,\middle|\,\widetilde{6sm_I}\right\rangle \Bigg|_{\substack{m_F=1/2 \\ m_I=1/2}}$$

PNC amplitude

$$E_{PNC}=\left\langle \widetilde{7sm_F}\left|d\right|\widetilde{6sm_I}\right\rangle\Bigg|_{\substack{m_F=1/2\\ m_I=1/2}}$$

$$E_{PNC}=-$$



$$\sum_{\mathrm{n}}\left\{\frac{\left\langle 7s\left\|d\right\|np_{1/2}\right\rangle\left\langle np_{1/2}\left\|H_{PNC}\right\|6s\right\rangle}{\varepsilon_{7s}-\varepsilon_{np_{1/2}}}\right.$$

$$\left.+\frac{\left\langle 7s\left\|H_{PNC}\right\|np_{1/2}\right\rangle\left\langle np_{1/2}\left\|d\right\|6s\right\rangle}{\varepsilon_{7s}-\varepsilon_{np_{1/2}}}\right\}$$

$$\begin{pmatrix} 1/2 & 1 & 1/2 \\ -1/2 & 0 & 1/2 \end{pmatrix}=\frac{1}{\sqrt{6}}$$

$$E_{PNC}=\frac{1}{\sqrt{6}}\sum_{\mathrm{n}}\left\{\frac{\left\langle 7s\left\|d\right\|np_{1/2}\right\rangle\left\langle np_{1/2}\left\|H_{PNC}\right\|6s\right\rangle}{\varepsilon_{7s}-\varepsilon_{np_{1/2}}}+\frac{\left\langle 7s\left\|H_{PNC}\right\|np_{1/2}\right\rangle\left\langle np_{1/2}\left\|d\right\|6s\right\rangle}{\varepsilon_{7s}-\varepsilon_{np_{1/2}}}\right\}$$

Note that we are considering the specific case of 6s-7s transition in Cs

How to calculate sum over n?

$$E_{PNC}=\frac{1}{\sqrt{6}}\sum_{n}\left\{\frac{\langle 7s\|d\|np_{1/2}\rangle\langle np_{1/2}\|H_{PNC}\|6s\rangle}{\varepsilon_{7s}-\varepsilon_{np1/2}}+\frac{\langle 7s\|H_{PNC}\|np_{1/2}\rangle\langle np_{1/2}\|d\|6s\rangle}{\varepsilon_{7s}-\varepsilon_{np1/2}}\right\}$$

How many n do we have to sum over? n=6, 7, 8, 9, 10, 11 …
The sum over n converges very fast, only first few terms are large but others still need to be estimated.

The strategy is the following:

1) Separate the sum into the MAIN term (n=6, 7, 8, 9) and the TAIL (n>9).
2) Introduce discrete basis set and, therefore, reduce infinite sum and integral over real spectrum to a finite sum over pseudospectrum (in a cavity). Make sure MAIN term states are real spectrum states (fit inside the cavity).
3) Calculate MAIN matrix elements very accurately (all-order method).
4) Calculate the TAIL as the sum over the pseudospectrum basis states using low-accuracy method.

See "Lectures on Atomic Physics" pages 173-176 about discrete basis sets

Parity nonconservation & weak charge Q_W

Nuclear density function

$$H^{(1)}_{PNC} = \frac{G_F}{2\sqrt{2}} Q_W \gamma_5 \rho(r)$$

Weak charge

G_F -Universal Fermi coupling constant

γ_5 - Dirac matrix

Q_W derived from the atomic PNC effects

Compare ?

If resulting Q_W deviates from $Q_W^{SM} \Rightarrow$ it indicates new physics beyond the Standard Model and sets limits on such theories

Standard Model prediction $Q_W^{SM} = -73.09(3)$

Why do we need atomic physics calculations to infer Q_W?

C.S. Wood et al. Science 275, 1759 (1997)

$$\frac{\mathrm{Im}\left(E_{\mathrm{PNC}}\right)}{\beta} = -1.5935(56)\,\frac{\mathrm{mV}}{\mathrm{cm}}$$

0.3% accuracy!!!

To infer $\boldsymbol{Q_W}$ we need:

1) Theoretical value of PNC amplitude E_{PNC} in terms of Q_W.

2) Either theoretical or experimental value of tensor transition polarizability β.

2.5σ story

-0.905 Blundell et al. (1992)
-0.908 Dzuba et al. (1989)

1997

1σ

- PNC amplitude theory accuracy: 1%
- Tensor transition polarizability β : theory value, 1%

$Q_W=-72.11(27)_{\mathbf{expt}}(89)_{\mathbf{theor}}$

Wood et al. (1997)

1999

2.5σ

- Measurement of β: 27.02(8) a_0^3
- New analyses of the theoretical accuracy
  (comparison of other properties with recent experiment)

0.4%

- Average of the two values, E_{PNC}=-0.9065(36), is used.

$Q_W=-72.06(28)_{\mathbf{expt}}(34)_{\mathbf{theor}}$

Bennett and Wieman (1999)

Q_W evolution …

Wood et al. (1997) $\mathrm{Im}(E_{\mathrm{PNC}})/\beta$	$-72.11(27)_{\mathrm{expt}}(89)_{\mathrm{theor}}$	1σ
Bennett & Wieman (1999) Measurement of β	$-72.06(28)_{\mathrm{expt}}(34)_{\mathrm{theor}}$	2.5σ
Derevianko (2000,2002) Calculation of Breit correction	$-72.61(28)_{\mathrm{expt}}(34/73)_{\mathrm{theor}}$	1.3σ/0.7σ
Dzuba et al. (2000) Calculation of Breit correction	$-72.42(28)_{\mathrm{expt}}(74)_{\mathrm{theor}}$	1.5σ/no dev.
Kozlov et al. (2001) Calculation of E_{PNC}, Breit correction	$-72.5(7)$	no deviation
Johnson et al. (2001) Calculation of vacuum pol. corr.	$-72.12(28)_{\mathrm{expt}}(34/74)_{\mathrm{theor}}$	2.2σ/1.2σ
Milstein & Sushkov (2002) Calculation of vacuum pol. corr.		2.2σ
Vasilyev et al. (2002) Measurement of 6s-7p trans., β	$-72.65(49)$	1.1σ
Dzuba et al. (2002) E_{PNC}	$-72.16(29)_{\mathrm{expt}}(36)_{\mathrm{th}}$	2σ
Flambaum & Kuchiev (2002)	$-72.71(29)_{\mathrm{expt}}(36)_{\mathrm{th}}$	no deviation
Milstein et al. (2003) self-energy & vertex corr.	$-72.81(28)_{\mathrm{expt}}(36)_{\mathrm{th}}$	0.6σ

Hyperfine interaction: interaction of the atomic electron with nucleus multipole moments.

Hyperfine structure: corresponding splitting of the energy levels.

Atomic angular momentum eigenstate $\left|J, M_J\right\rangle$

Nuclear angular momentum eigenstate $\left|I, M_I\right\rangle$

$\left|IJ, FM_F\right\rangle$

$\boxed{F=J+I.}$

$$H_{hfs} = \sum_{k\lambda} (-1)^{\lambda} \underbrace{\mathrm{T}^{k}_{\lambda}}_{\text{Acts on electronic coordinates and spin}} \underbrace{\mathrm{T}^{k}_{-\lambda}}_{\text{Acts on nuclear coordinates and spin}}$$

Example: ^{133}Cs
J=1/2, I=7/2
F=3,4

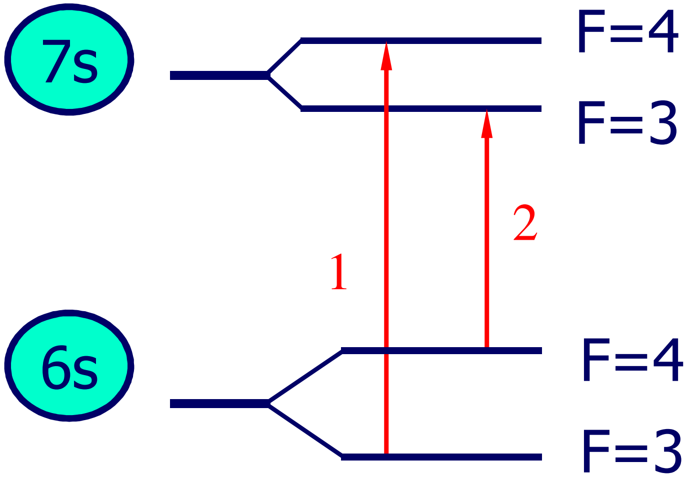
k=1: Magnetic-dipole interaction
Hyperfine constant A

$$A = \frac{g_I}{j_v}\left\langle n_v k_v m_v = j_v \left| t_0^1 \right| n_v k_v m_v = j_v \right\rangle \times 13075 MHz$$

$$\Delta E(JIF) = \tfrac{1}{2} A\left[F(F+1) - J(J+1) - I(I+1)\right]$$

Spin-dependent PNC effects

^{133}Cs J=1/2, I=7/2, F=3,4



$$\frac{\mathrm{Im}\left(E_{\mathrm{PNC}}\right)}{\beta} = \begin{cases} -1.6349(80)\ {}^{\mathrm{mV}}\!/_{\mathrm{cm}} & 1 \\ -1.5576(77)\ {}^{\mathrm{mV}}\!/_{\mathrm{cm}} & 2 \end{cases}$$

PNC amplitude depends on the nuclear spin!

$$\Delta\left[\mathrm{Im}\left(E_{\mathrm{PNC}}^{\mathrm{sd}}\right)/\beta\right]_{34-43} = -0.077(11)\ {}^{\mathrm{mV}}\!/_{\mathrm{cm}}$$

C.S. Wood et al. Science 275, 1759 (1997)

PNC & hyperfine states

$F=J+I.$

Let go back to calculation of nuclear-spin independent PNC first and evaluate matrix element between hyperfine states

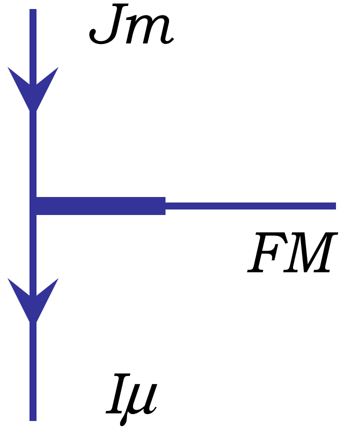
Example: ^{133}Cs
J=1/2, I=7/2
F=3,4

Atomic angular momentum eigenstate $|Jm\rangle$

Nuclear angular momentum eigenstate $|I\mu\rangle$

$$|IJ, FM\rangle$$

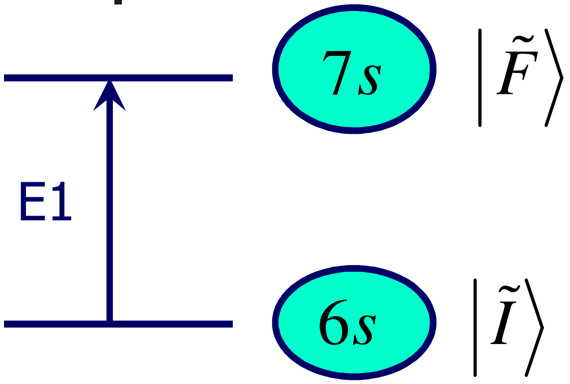
$$|FM\rangle = \sum_{m\mu} -$$



$$|Jm\rangle |I\mu\rangle$$

Cs

Quantum numbers?



$$\langle \tilde{F}|d|\tilde{I}\rangle = \sum_{N\neq F} \frac{\langle F|H_{PNC}|N\rangle\langle N|d|I\rangle}{\varepsilon_F - \varepsilon_N} + \sum_{N\neq I} \frac{\langle F|d|N\rangle\langle N|H_{PNC}|I\rangle}{\varepsilon_I - \varepsilon_N}$$

What quantum numbers describe $|F\rangle$, $|I\rangle$, and $|N\rangle$?

Our PNC Hamiltonian is nuclear-spin independent
so we can take only $|n\, l\; j\, m_j\rangle$
l is included to indicate parity of the state.

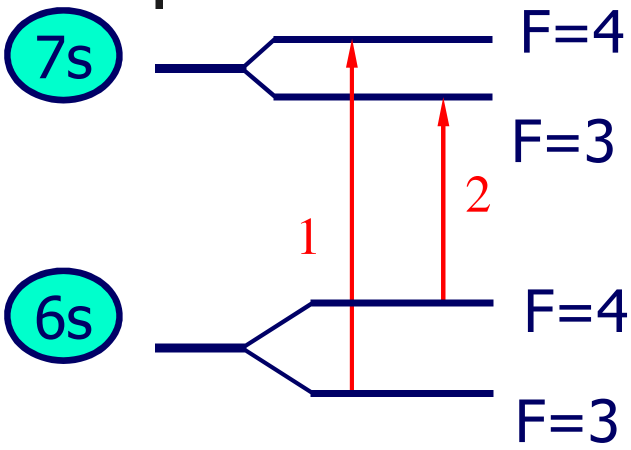
$$|F\rangle = |n=7\, l=0\; j=1/2\, m_F\rangle = |7\mathrm{s}\; m_F\rangle$$
$$|I\rangle = |n=6\, l=0\; j=1/2\, m_I\rangle = |6\mathrm{s}\; m_I\rangle$$
$$|N\rangle = |n\, l\; j\, m_N\rangle$$

Note s state has
only j=1/2
so we write

$$6s \equiv 6s_{1/2}$$
$$7s \equiv 7s_{1/2}$$

Cs

Quantum numbers?

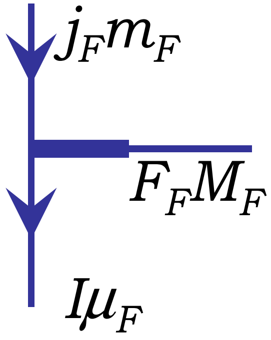


$$\langle \tilde{F} | d | \tilde{I} \rangle =$$

$$\sum_{N \neq F} \frac{\langle F | H_{PNC} | N \rangle \langle N | d | I \rangle}{\varepsilon_F - \varepsilon_N} + \sum_{N \neq I} \frac{\langle F | d | N \rangle \langle N | H_{PNC} | I \rangle}{\varepsilon_I - \varepsilon_N}$$

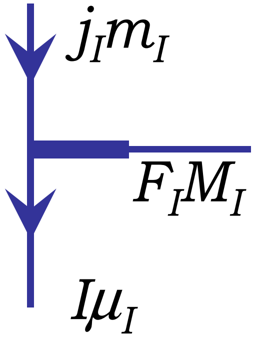
What quantum numbers describe $|F\rangle$, $|I\rangle$, and $|N\rangle$?

$$|F\rangle = |(j_F I) F_F M_F\rangle = \sum_{m_F \mu_F} -$$



$$|j_F \; m_F I \mu_F\rangle$$

$$|I\rangle = |(j_I I) F_I M_I\rangle = \sum_{m_I \mu_I} -$$



$$|j_I \; m_I I \mu_I\rangle$$

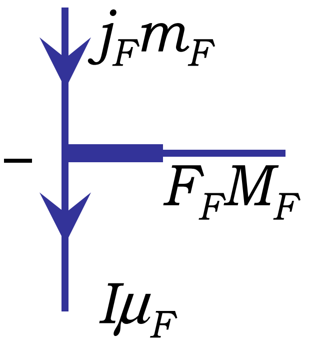
$$|N\rangle = |j_n \; m_n I \mu_n\rangle$$

Cs

Term 1

$$\text{Term 1} = \sum_{N \neq F} \frac{\langle F\,|H_{PNC}|\,N\rangle\langle N\,|d|\,I\rangle}{\varepsilon_F - \varepsilon_N} = \sum_{\substack{m_I\,\mu_I\\ m_F\,\mu_F}} \sum_{\substack{n j_n l_n\\ m_n\,\mu_n}}$$

−



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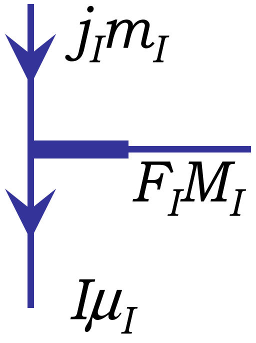
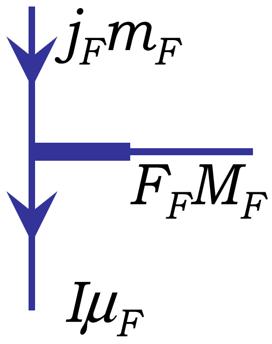


$$\frac{\langle j_F m_F I \mu_F\,|H_{PNC}|\,j_n m_n I \mu_n\rangle\langle j_n m_n I \mu_n\,|d|\,j_I m_I I \mu_I\rangle}{\varepsilon_F - \varepsilon_N}$$

$$= \sum_{\substack{m_I\,\mu_I\\ m_F\,\mu_F}} \sum_{\substack{n j_n l_n\\ m_n\,\mu_n}}$$

−



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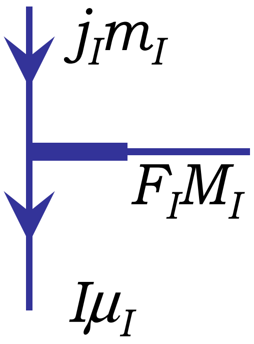

$$\frac{\langle j_F m_F\,|H_{PNC}|\,j_n m_n\rangle\,\delta_{\mu_F \mu_n}\,\langle j_n m_n\,|d|\,j_I m_I\rangle\,\delta_{\mu_I \mu_n}}{\varepsilon_F - \varepsilon_N}$$

$$= \sum_{\substack{m_I\,\mu_I\\ m_F\,\mu_F}} \sum_{\substack{n j_n l_n\\ m_n}}$$

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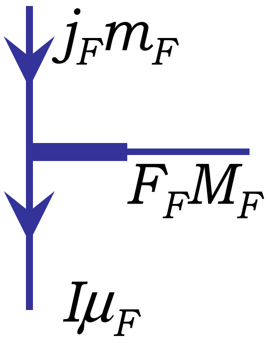

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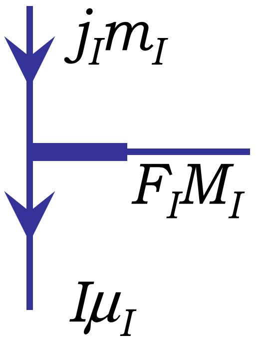

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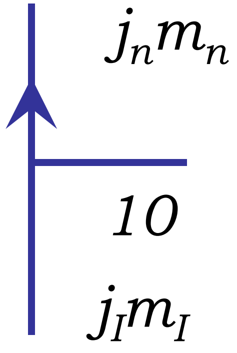
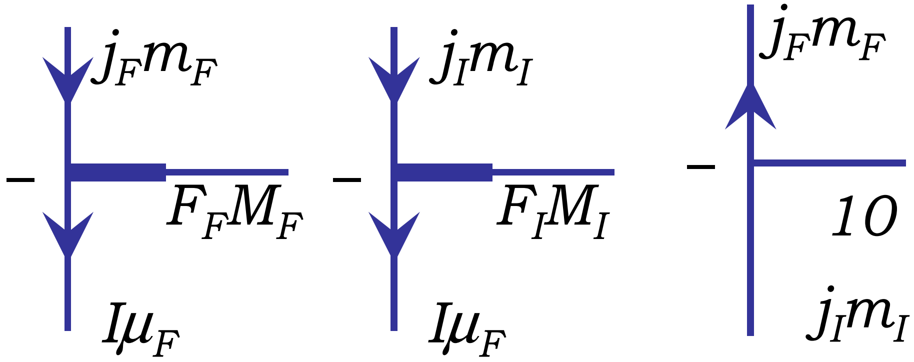


$$\frac{\langle j_F \|H_{PNC}\| j_n\rangle\,\delta_{j_F j_n}\,\delta_{\mu_F \mu_I}\,\langle j_n \|d\| j_I\rangle}{\varepsilon_F - \varepsilon_N}$$

Cs

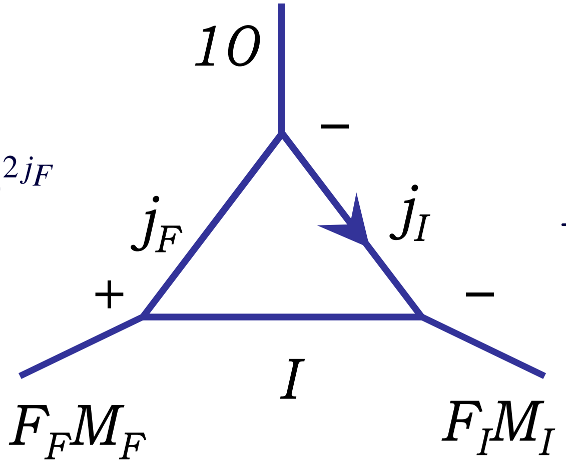
Term 1

$$= \sum_{\substack{m_I \\ m_F\ \mu_F}} \sum_n$$



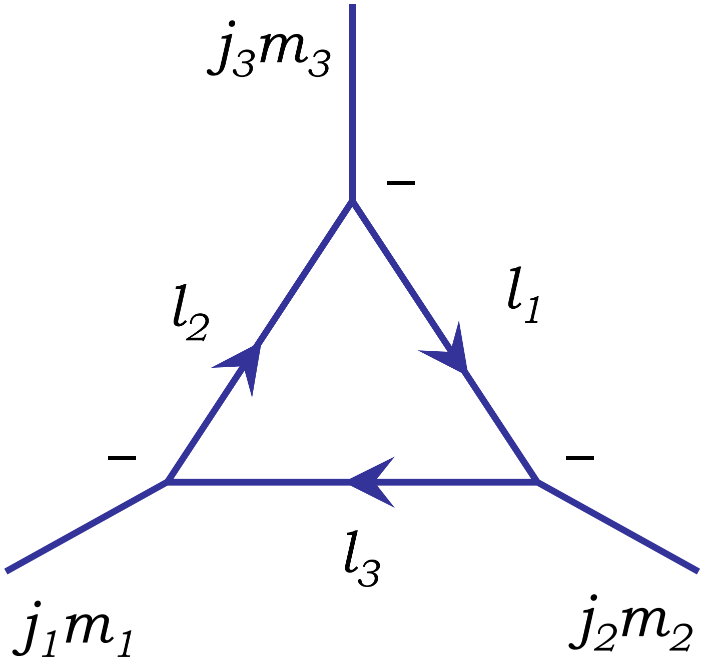
$$\frac{\left\langle j_F \middle\| H_{PNC} \middle\| j_n \right\rangle \left\langle j_n \middle\| d \middle\| j_I \right\rangle}{\varepsilon_F - \varepsilon_N}$$

$$= \sum_n \sqrt{(2F_F+1)(2F_I+1)}(-1)^{2j_F}$$

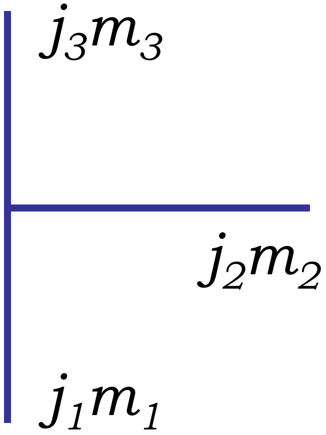


$$\frac{\left\langle j_F \middle\| H_{PNC} \middle\| j_n \right\rangle \left\langle j_n \middle\| d \middle\| j_I \right\rangle}{\varepsilon_F - \varepsilon_N}$$

6-j symbol & how to remove triangle from the graph



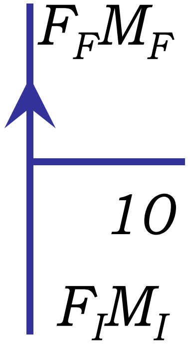
$$= \begin{Bmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{Bmatrix} \quad +$$



Cs

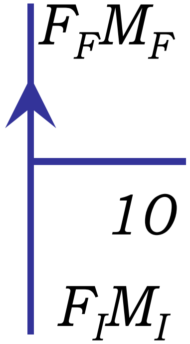
Result

$$=\sum_n \sqrt{(2F_F+1)(2F_I+1)}(-1)^{j_F+I+F_I+1}\begin{Bmatrix} F_F & F_I & 1 \\ j_I & j_F & I \end{Bmatrix} -$$



$$\frac{\langle j_F\|H_{PNC}\|j_n\rangle\langle j_n\|d\|j_I\rangle}{\varepsilon_F-\varepsilon_N}$$

$$\langle\tilde{F}|d|\tilde{I}\rangle=\langle\widetilde{F_FM_F}|d|\widetilde{F_IM_I}\rangle=\sum_n\sqrt{[F_F][F_I]}(-1)^{j_F+I+F_I+1}\begin{Bmatrix} F_F & F_I & 1 \\ j_I & j_F & I \end{Bmatrix} -$$



$$\times\sum_{\mathrm{n}}\left\{\left.\frac{\langle j_F\|H_{PNC}\|j_n\rangle\langle j_n\|d\|j_I\rangle}{\varepsilon_F-\varepsilon_n}\right|_{\pi_n=-\pi_F}+\left.\frac{\langle j_F\|d\|j_n\rangle\langle j_n\|H_{PNC}\|j_I\rangle}{\varepsilon_I-\varepsilon_n}\right|_{\pi_n=-\pi_I}\right\}$$

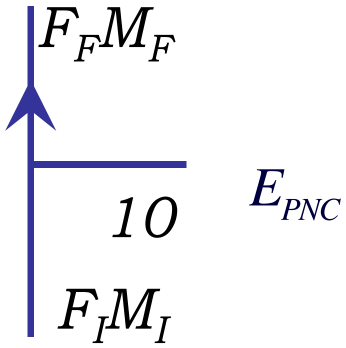
Cs

Result

$$\left\langle \tilde{F} \middle| d \middle| \tilde{I} \right\rangle = \left\langle \widetilde{F_F M_F} \middle| d \middle| \widetilde{F_I M_I} \right\rangle = \sum_n \sqrt{[F_F][F_I]}(-1)^{3/2+I+F_I} \begin{Bmatrix} F_F & F_I & 1 \\ 1/2 & 1/2 & I \end{Bmatrix}$$

$$\times \sum_{\mathrm{n}} \left\{ \frac{\langle 7s \| H_{PNC} \| np_{1/2} \rangle \langle np_{1/2} \| d \| j_I \rangle}{\varepsilon_{7s} - \varepsilon_{np_{1/2}}} + \frac{\langle 7s \| d \| np_{1/2} \rangle \langle np_{1/2} \| H_{PNC} \| 6s \rangle}{\varepsilon_{6s} - \varepsilon_{np_{1/2}}} \right\}$$

$$= \sum_n \sqrt{6[F_F][F_I]}(-1)^{3/2+I+F_I} \begin{Bmatrix} F_F & F_I & 1 \\ 1/2 & 1/2 & I \end{Bmatrix} \; -$$



We derived the corresponding matrix element between the hyperfine states!

Back to nuclear spin-dependent parity nonconserving effects

Nuclear anapole moment

Parity-violating nuclear moment

Valence nucleon density

$$H^{(\mathrm{a})}_{\mathrm{PNC}} = \frac{G_F}{\sqrt{2}} \boxed{\kappa_a}\ \boldsymbol{\alpha}\bullet\mathbf{I}\ \ \rho_v(r)$$

How to derive the value of the nuclear anapole moment?

Research project/current homework

$$H^{\mathrm{(a)}}_{\mathrm{PNC}} = \frac{G_F}{\sqrt{2}} \; \kappa_a \; \boldsymbol{\alpha} \bullet \mathbf{I} \; \rho_v(r)$$

This Hamiltonian may be decomposed to:

$$H^{\mathrm{(a)}}_{\mathrm{PNC}} = \sum_{\mu} (-1)^{\mu} I_{-\mu} H^{(a)}_{\mu}$$

Acts on nuclear coordinates and spin

Acts on electron coordinates and spin

$$\left\langle j_F m_F I \mu_F \middle| H^{(a)}_{PNC} \middle| j_n m_n I \mu_n \right\rangle \to \sum_{\mu} (-1)^{\mu} \left\langle j_F m_F \middle| H^{(a)}_{\mu} \middle| j_n m_n \right\rangle \left\langle I \mu_F \middle| I_{-\mu} \middle| I \mu_n \right\rangle$$

Problem: calculate the corresponding normally forbidden electric-dipole <F|d|I> matrix element which is non-zero owing to the PNC interaction above.

Hint: follow the example given in class.

Nuclear anapole moment ?

The constraints obtained from the Cs experiment were found to be inconsistent with constraints from other nuclear PNC measurements, which favor a smaller value of the ^{133}Cs anapole moment.