Lecture 10

Interaction of atoms with external fields. Zeeman effect.

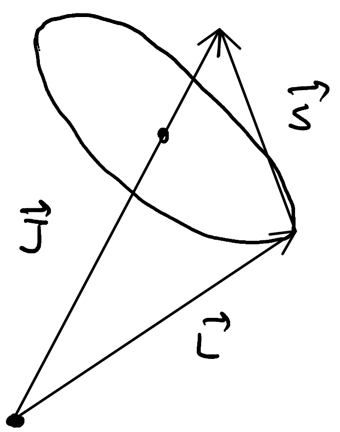
In 1896 P. Zeeman observed that the spectral lines of atoms were split in the presence of the external magnetic field.

The atom's magnetic moment has orbital and spin contributions: (we use $\hbar=1$ below)

$$\vec{\mu} = -\mu_B \vec{L} - g_s \mu_B \vec{S}$$

The interaction of the atom with an external field is described by $H_{ZE} = -\vec{\mu}\cdot\vec{B}$

The expectation value of such Hamiltonian can be calculate in the basis $|LSJM_J\rangle$ if $E_{ZE} \ll E_{s\text{-}o} \ll E_{re}$. In this case, the interaction of atom with the magnetic field can be treated as a perturbation to the fine-structure levels. Using the vector model, we consider that in the presence of the spin-orbit coupling, L and S precess about the fixed total angular momentum J:



Their corresponding (time) averaged values are their projection along J:

$$\vec{S}_{ave} = \frac{(\vec{S}\cdot\vec{J})}{|J|^2}\vec{J} \qquad \vec{L}_{ave} = \frac{(\vec{L}\cdot\vec{J})}{|J|^2}\vec{J}$$

Therefore, we get

$$H_{ZE} = -\frac{\langle\vec{\mu}\cdot\vec{J}\rangle}{J(J+1)}\vec{J}\cdot\vec{B} =$$

taking $[\vec{B} = B\hat{e}_z]$

$$= \frac{\langle\vec{L}\cdot\vec{J}\rangle + g_s\langle\vec{S}\cdot\vec{J}\rangle}{J(J+1)}\mu_B B J_z$$

In the quantum treatment, the quantities in the brackets are expectation values in the form $\langle JM_J|\ldots|JM_J\rangle$. Therefore, we get

$$E_{ZE} = g_J \mu_B B M_J$$

$\uparrow$ Landé g-factor

$$g_J = \{ \langle \vec{L}\cdot\vec{J} \rangle + g_s \langle \vec{S}\cdot\vec{J} \rangle \} / J(J+1)$$

Since $g_s \approx 2 \Rightarrow \langle \vec{L}\cdot\vec{J} \rangle + 2\langle \vec{S}\cdot\vec{J} \rangle$

$\vec{L} = \vec{J} - \vec{S} \Rightarrow$

$\bar{L}^2 = \bar{J}^2 + \bar{S}^2 - 2\bar{J}\cdot\bar{S} \Rightarrow \vec{J}\cdot\vec{S} = \frac{1}{2}(-\bar{L}^2 + \bar{J}^2 + \bar{S}^2)$

$\vec{S} = \vec{J} - \vec{L}$

$\bar{S}^2 = J^2 + \bar{L}^2 - 2\cdot\vec{J}\cdot\vec{L} \Rightarrow \vec{J}\cdot\vec{L} = \frac{1}{2}(-\bar{S}^2 + \bar{J}^2 + \bar{L}^2)$

$$\langle \vec{L}\cdot\vec{J} \rangle + 2\langle \vec{S}\cdot\vec{J} \rangle = \frac{1}{2}(-S(S+1) + J(J+1) + L(L+1))$$
$$-L(L+1) + S(S+1) + J(J+1)$$

$$= \frac{1}{2}S(S+1) - \frac{1}{2}L(L+1) + \frac{3}{2}J(J+1)$$

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

<u>Zeeman effect and hyperfine structure</u>

The total atomic magnetic moment is the sum of the electronic and nuclear moments:

$$\vec{\mu}_{atom} = -g_J \mu_B \vec{J} + g_I \mu_N \vec{I} \simeq -g_J \mu_B \vec{J}$$

but we can neglect the nuclear contribution for most cases since $\mu_N \ll \mu_B$ (due to m_e/m_p mass ratio)

Then, the interaction Hamiltonian is $H = g_J \mu_B \vec{J}\cdot\vec{B}$

While this interaction does not depend on the nuclear spin, the expectation values depends on the hyperfine structure. We consider the weak-field and strong-field cases.

Zeeman effect of a weak field ($\mu_B B < A$)

In the weak-field regime, the interaction with the external field is weaker than $A\,\vec{I}\cdot\vec{J}$ so it can be treated as a perturbation to the hyperfine structure. In this regime, F and M_F are good quantum numbers, but not M_I and M_J. Taking the projection of the magnetic moments along F gives

$$H = g_J \mu_B \frac{\langle \bar{J}\cdot\bar{F}\rangle}{F(F+1)} \vec{F}\cdot\vec{B} = g_F \mu_B \vec{F}\cdot\vec{B} = g_F \mu_B B F_z$$

where

$$g_F = \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} g_J$$

The corresponding Zeeman energy shift is

$$E = g_F \mu_B B M_F$$

Zeeman effect of a strong field ($\mu_B B < A$)

If the interaction with the external field is greater than $|I M_I J M_J\rangle$, F is not a good quantum number. The effect of the hyperfine interaction can be calculated as a perturbation on the eigenstates,

$$E_{ZE} = g_J \mu_B B M_J + \langle I M_I J M_J | A\,\vec{I}\cdot\vec{J} | I M_I J M_J \rangle$$

$$= g_J \mu_B B M_J + A M_I M_J$$

$$\vec{I}\cdot\vec{J} = \underbrace{I_x J_x + I_y J_y}_{} + I_z J_z$$

x and y components average to zero in the precession about the field along z-direction

Example: Zeeman effect in hydrogen hyperfine levels