Lecture 22

Summary

The equations for the **c_a and c_b coefficients** ($H'_{ii} = 0$)

$$\dot{c}_a = -\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t} c_b$$

$$\dot{c}_b = -\frac{i}{\hbar} H'_{ba} e^{i\omega_0 t} c_a$$

First-order perturbation theory gives

$$c_b^{(1)} = -\frac{i}{\hbar} \int_0^t H'_{ba}(t') e^{i\omega_0 t'} dt'$$

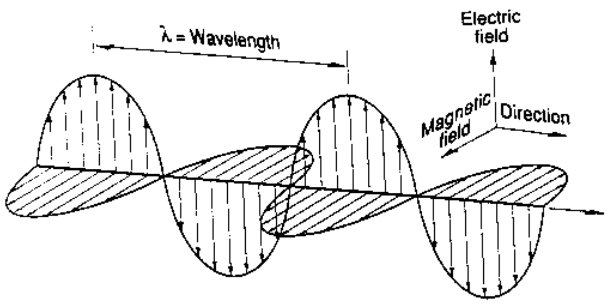
Sinusoidal perturbation

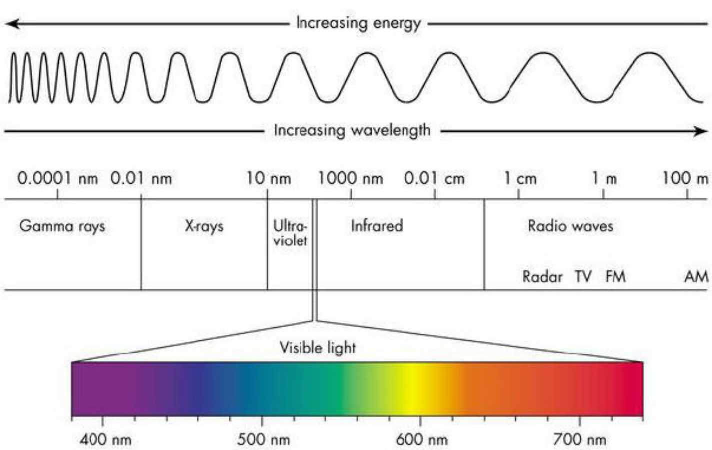
$$H'(\vec{r}, t) = V(\vec{r}) \cos(\omega t)$$

$$P_{a \to b}(t) = |c_b(t)|^2 \cong \frac{|V_{ba}|^2}{\hbar^2} \frac{\sin^2\{(\omega_0 - \omega)t/2\}}{(\omega_0 - \omega)^2}$$

Emission and absorption of radiation

Electromagnetic waves





An atom respond primarily to electric component. Comparing the atomic dimensions (Bohr radius that is atomic unit of length is a_0=0.05 nm, so even 40a_0 is only 0.2 nm) with wavelength of visible light (see above), we see that we can ignore the spatial variation of the field.

Therefore, we write the oscillating electric field as

$$\bar{E} = E_0 \cos(\omega t)\,\hat{k}.$$

We assume here that light is monochromatic (i.e. of the same wavelength) and polarized along z direction.

The corresponding perturbing Hamiltonian is

electric charge

$$H' = -qE_0 \cos(\omega t)\, z$$

Therefore, our matrix element H' is given by

electric-dipole moment

$$H'_{ba} = -\wp E_0 \cos(\omega t); \qquad \wp \equiv q\langle \psi_a | z | \psi_b \rangle.$$

Our assumption that diagonal matrix elements of H' are zero is valid here, since ψ is either odd or even function, therefore $z|\psi|^2$ is odd function and the corresponding integral is zero. Therefore, we can use our result derived for the sinusoidal perturbation with the matrix element of V being

$$V_{ba} = -\wp E_0$$

Absorption, stimulated emission, and spontaneous emission

Our atom starts in the lower state a. According to the first-order formula that we have derived during the last lecture, the probability of the transition to upper level b due to the monochromatic light wave is given by

$$P_{a\to b}(t) = \left(\frac{|\wp| E_0}{\hbar}\right)^2 \frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}.$$

The energy absorbed by the atom is

$$E_b - E_a = \hbar\omega_0.$$

If we repeat the derivation for the atom initially in the upper state b, i.e.

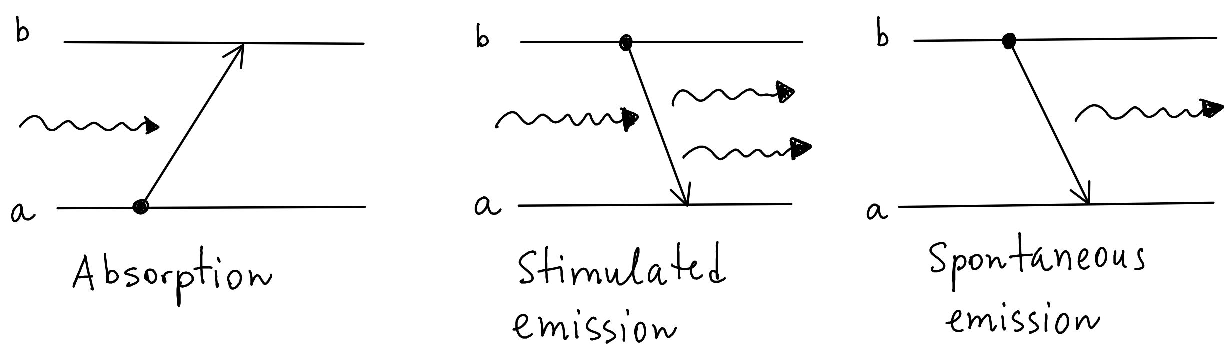
$$c_b(0) = 1; \quad c_a(0) = 0,$$

we find that the probability of the transition to the lower level is EXACTLY THE SAME:

$$P_{b\to a}(t) = \left(\frac{|\wp| E_0}{\hbar}\right)^2 \frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}$$

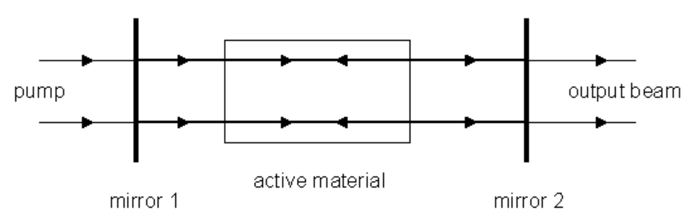
This is a remarkable result! If you shine light of the appropriate wavelength to the atom in the upper state, it can make transition to lower state. Then, the laser field GAINS the energy $\hbar\omega_0$ from the atom. This process is called stimulated emission. The probability of such transition is exactly the same as the probability of the excitation of the atom by the laser field from lower level a to upper level b.

Atom-light interactions:



The possibility to produce laser field energy gain makes amplification possible. If we have a large number of atoms, than one photon can trigger a "chain reaction" when first photon will produce 2, these 2 produce 4, and so on. In fact, this is the principle of the operation of the laser.

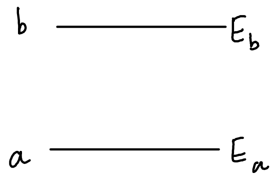
Light
Amplification by
Stimulated
Emission of
Radiation



The only problem in this scheme is that you need most of the atoms to be in the upper state which is normally not the case! So you need to somehow create the **population inversion**.

How do lasers work?

See simulation "Lasers" at http://phet.colorado.edu/simulations/sims.php?sim=Lasers



Let us first demonstrate why one needs to achieve population inversion for laser action. Suppose we have for now two-level system (as in our previous derivation) with energies E_a and E_b. A beam of the electromagnetic radiation of intensity I and angular frequency $\omega = (E_b - E_a)/\hbar$ is passing through the material.

The rate of change of the average energy density because of the absorption from the beam is

$$\frac{d\rho_a}{dt} = -\underbrace{N_a}_{\text{number of atoms in level } a}\hbar\omega \overbrace{W_{ab}}^{\text{transition rate per atom per absorption}}.$$

The rate of change of the average density due to stimulated emission is

$$\frac{d\rho_s}{dt} = \underbrace{N_b}_{\text{number of atoms in level } b}\hbar\omega \overbrace{W_{ba}}^{\text{transition rate per atom per absorption}} \qquad W_{ab} = W_{ba}$$

The cross-section defined as

$$\sigma = \frac{\hbar\omega W_{ab}}{I} \qquad (\text{Note that } W_{ab} \propto \underbrace{I}_{\text{intensity}})$$

depend on the level properties but does not depend on the intensity of the beam.

Net change of the average energy per unit volume transversed by the beam is given by

$$\frac{d\rho}{dt} = \frac{d\rho_a}{dt} + \frac{d\rho_s}{dt} = (N_2 - N_1)\hbar\omega W_{ab} = (N_2 - N_1)\sigma I$$

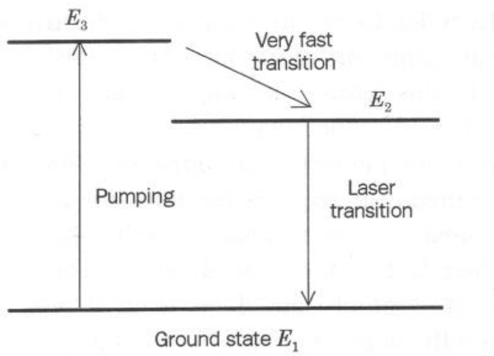
If the beam is travelling parallel to the z axis then we have $\quad (\text{since } I = \rho c)$

$$\frac{dI}{dz} = \frac{d\rho}{dt} = \sigma I (N_2 - N_1)$$

Therefore, we see that if $N_1 > N_2$ the incident radiation is absorbed as it travels through the material. If $N_2 > N_1$, then the radiation is amplified. The next question is how to achieve population inversion.

How to create population inversion:

Three-level scheme



Four-level scheme