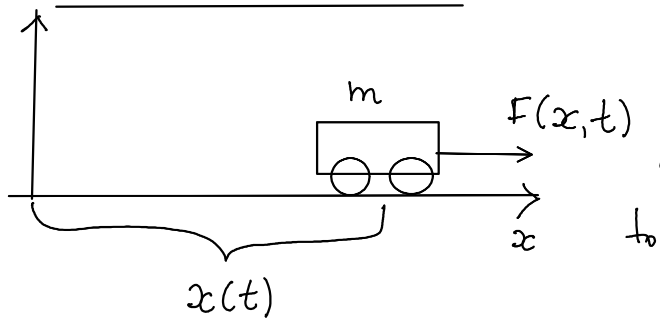
Lecture 1

Classical mechanics:



Particle of mass m, constrained to move along x-axis, subject to some force $F(x,t)$.

Task of classical mechanics: find $x(t)$.

If we find $x(t)$, we can find velocity $v = dx/dt$, momentum $p = mv$, kinetic energy $T = \frac{1}{2}mv^2$, and so on.

How do we determine $x(t)$? Use second Newton's Law $F_x = ma_x$; for conservative forces $F_x = -\frac{\partial V}{\partial x}$ =>

$F_x = ma_x$ => $\boxed{m\frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x}}$ + initial conditions,

generally position and velocity at $t=0$.

Quantum mechanics

Task: we want to determine particle's $\boxed{\text{wave function } \psi}$.

To do so, we use Schrödinger equation:

$$\boxed{i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi}$$ + initial conditions

$\hbar$ is a Plank's constant (or actually $\frac{h}{2\pi}$).

$$\hbar = \frac{h}{2\pi} = 1.054572 \times 10^{-34} \text{ Js.}$$

Note: wave function is complex, but $\Psi^*\Psi$ is real and nonnegative. Ψ^* is a complex conjugate of Ψ.

So, we can find the wave function.

$\boxed{\text{What is the wave function?}}$

Born's statistical interpretation of the wave function:

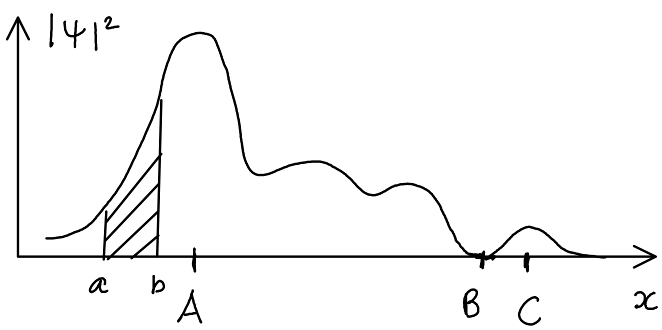
$|\Psi(x,t)|^2$ gives the probability of finding the particle at the point x, at time t.

More precisely,

$$\int_a^b |\Psi(x,t)|^2 dx = \left\{ \begin{array}{l} \text{probability of finding} \\ \text{a and b, at time t} \end{array} \right\}$$

Problem: indeterminacy of the quantum mechanics. Even if you know everything that theory (i.e. quantum mechanics) has to tell you about the particle (i.e. wave function), you can not predict with certainty where this particle is going to be found by the experiment.

Quantum mechanics provides statistical information about possible results.



Example: particle is likely to be found in the vicinity of A and is unlikely to be found in the vicinity of B.

Now, suppose we make a measurement and find particle at C.

Question: where was the particle just before the measurement ?

Answer # 1. Realist position.

It was at C. That means quantum mechanics is incomplete theory. Why?
Well, the particle was at C, but quantum mechanics could not predict it.
Therefore, Ψ does not give the whole story and we need additional information (hidden variables) to provide a complete description of the particle.

Answer #2. The orthodox position.

The particle was not really anywhere. It was an act of measurement that forced particle to "take a stand". We still have no idea why it "decided" on point C.
Note: there is something very strange about concept of measurement.

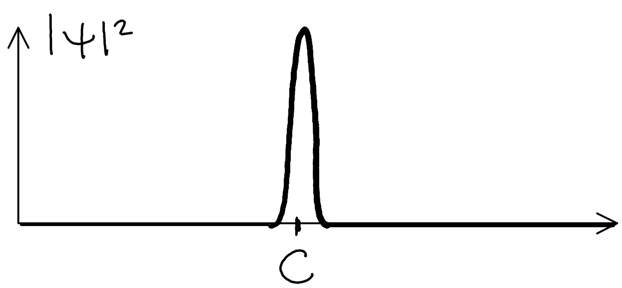
Answer #3. The agnostic position.

Refuse to answer. Since the only way to know if you were right is to make a measurement, you no longer get "before the measurement". Therefore, it can not be tested.

In 1964, Bell shown that it makes an observable difference if the particle has a precise (but unknown) position before measurement, which rules out answer #3.

What if we make a second measurement after the first?

Repeated measurement returns the same value.



The first measurement alters the wave function and it collapses to a spike at C.
After that, it will start evolving according to Schrödinger equation.

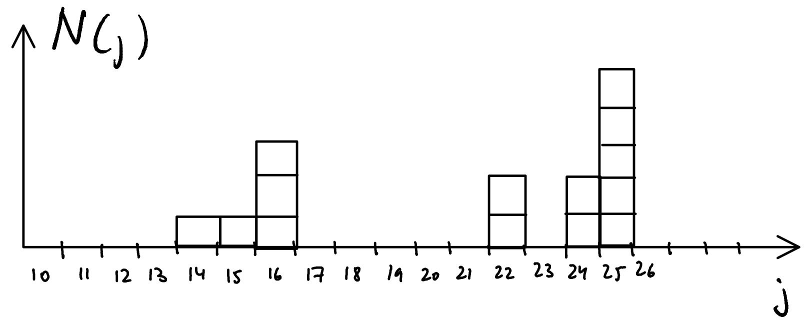
Note: $$\boxed{\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1}$$ "particle must be somewhere".

Probability

Discrete variables

EXAMPLE #1

N is the age of person in the room.



$N(14) = 1 \qquad N(22) = 2$

$N(15) = 1 \qquad N(24) = 2$

$N(16) = 3 \qquad N(25) = 5$

Total number of people: $N = \sum_{j=0}^{\infty} N(j)$.

1.Probability of person being of certain age?

$$P(j) = \frac{N(j)}{N}, \text{ so } \quad P(14) = \frac{1}{14}, \quad P(15) = \frac{1}{14}, \quad P(16) = \frac{3}{14}, \ldots$$

$$\sum_{j=0}^{\infty} P(j) = 1$$

2. Most probable age? $\boxed{25}$

3. What is the median age? $\boxed{23}$ (7 people are younger & 7 people are older).

4. Average (or mean) age?

Average value of j is designated $\langle j \rangle$.

$$\langle j \rangle = \frac{\sum j N(j)}{N} = \sum_{j=0}^{\infty} j P(j)$$

$$\langle j \rangle = 14 \cdot \underset{1/14}{P(14)} + 15 * \underset{1/14}{P(15)} + 16 * \underset{3/14}{P(16)} + \dots =$$

$$= \frac{294}{14} = 21$$

In quantum mechanics, the average is usually quantity of interest; in this context it is called **expectation value**.

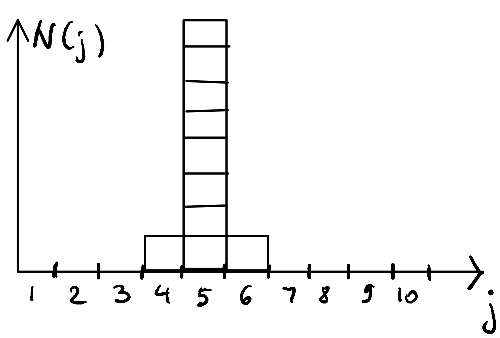
5. What is the average of the squares of ages?

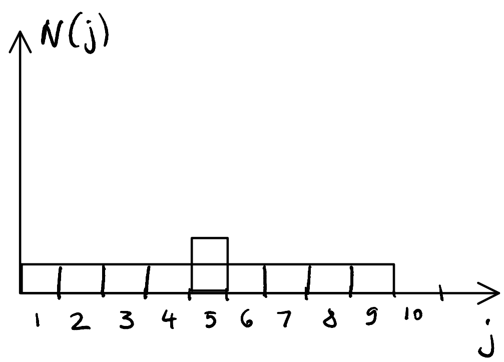
$$\left.\begin{array}{l} 14^2 = 196 \quad \text{with} \quad P = 1/14 \\ 16^2 = 256 \quad \text{with} \quad P = 3/14 \\ \vdots \end{array}\right\} \Rightarrow \langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j)$$

In general, the average value of some function of j is given by

$$\boxed{\langle f(j) \rangle = \sum_{j=0}^{\infty} f(j) P(j)}$$

Variance of the distribution





These two histograms have the same median, average, and the same most probable value, but different **standard deviations**.

σ: standard deviation, measure of the spread about $\langle j \rangle$: $\boxed{\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}}$

Continuous variables

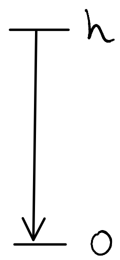
The probability that x lies between a and b (a finite interval) is:

$$P_{ab} = \int_a^b \rho(x)\,dx \qquad \rho(x) \text{ is a } \underline{\text{probability density.}}$$

$$\boxed{\begin{aligned} &\int_{-\infty}^{\infty} \rho(x)\,dx = 1 \\ &\langle x \rangle = \int_{-\infty}^{\infty} x\,\rho(x)\,dx \\ &\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)\,\rho(x)\,dx \\ &\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 \end{aligned}}$$

EXAMPLE #2

Suppose I drop a rock of a cliff of height h. As it falls, I snap a million photographs, at random intervals. On each picture, I measure the distance the rock has fallen.
Question: what is the average of all distances?



$x(t) = \frac{1}{2} g t^2$ (no air resistance); $\frac{dx}{dt} = gt$

Total flight time T is calculated from

$$h = \frac{1}{2} g T^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$$

Probability that camera flashes in the interval dt is $\frac{dt}{T} \Rightarrow$ probability that certain photograph shows a distance dx is:

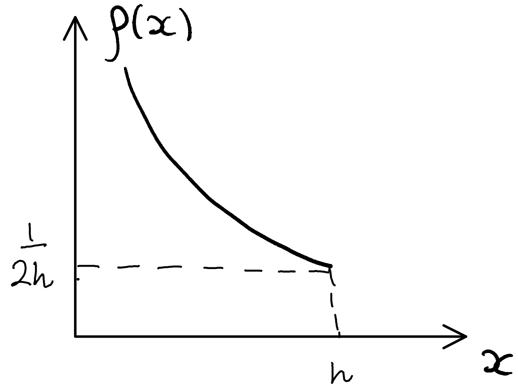
$\frac{dx}{dt} = gt$, $dt = dx/gt$

$T = \sqrt{\frac{2h}{g}}$

$$\frac{dt}{T} = \frac{dx}{gt}\sqrt{\frac{g}{2h}} = \frac{dx}{g\sqrt{2x}}\,\frac{\sqrt{g}\sqrt{g}}{\sqrt{2h}} = \frac{1}{2\sqrt{xh}}\,dx$$

$t = \sqrt{\frac{2x}{g}}$

$\rho(x)$, probability density for $0 \le x \le h$



Let's check if $\int_{-\infty}^{\infty} \rho(x)dx = 1$:

$$\int_0^h \frac{1}{2\sqrt{hx}}\,dx = \frac{1}{2\sqrt{h}}\left(2\sqrt{x}\right)\Big|_0^h = 1$$

Note: $\boxed{\int_a^b x^n\,dx = \frac{x^{n+1}}{n+1}\Big|_a^b \text{ if } n \neq -1}$

The average distance is:

$$\langle x \rangle = \int_0^h x\,\rho(x)\,dx = \int_0^h x\,\frac{1}{2\sqrt{hx}}\,dx = \frac{1}{2\sqrt{h}}\left(\frac{2}{3}x^{3/2}\Big|_0^h\right)$$

$$= \frac{\cancel{2}}{3\cdot\cancel{2}\sqrt{h}}\,h^{3/2} = \frac{h}{3}$$